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# Introduction to CMB Science

Paddy Leahy

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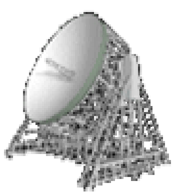

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# Contents

- Uniqueness of CMB science
- Basic CMB Phenomenology
- Origin of CMB fluctuations
- CMB polarization
- Secondary anisotropies: S-Z and lensing
- Technical issues
- Foregrounds
- Current & future experiments

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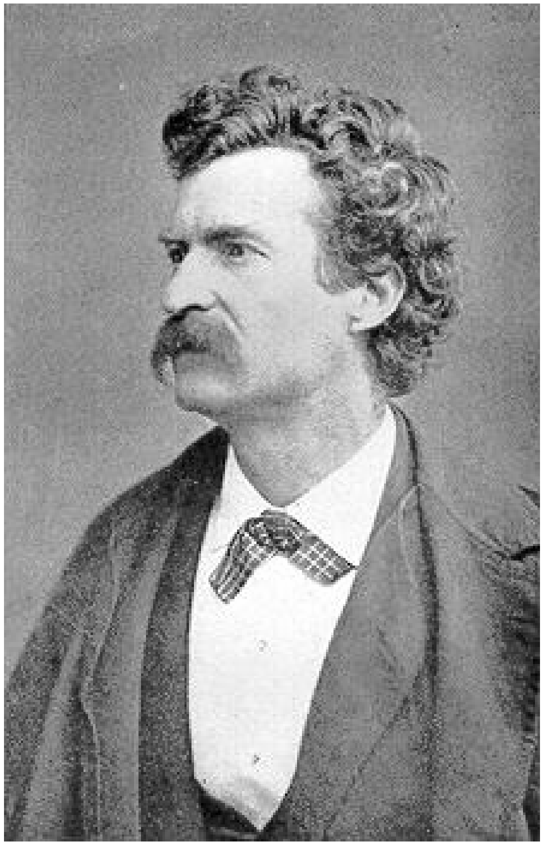
# Uniqueness of CMB Science



There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.

— Mark Twain, *Life on the Missisipi*

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# CMB Answers the Big Questions

- Direct test of geometry of the universe
- Reveals nature of primary fluctuations that lead to structure in the Universe
- Best measure of baryon content
- Best measure of non-baryonic content
- Multiple consistency checks on standard cosmological theory
- Future test of energy scale of inflation

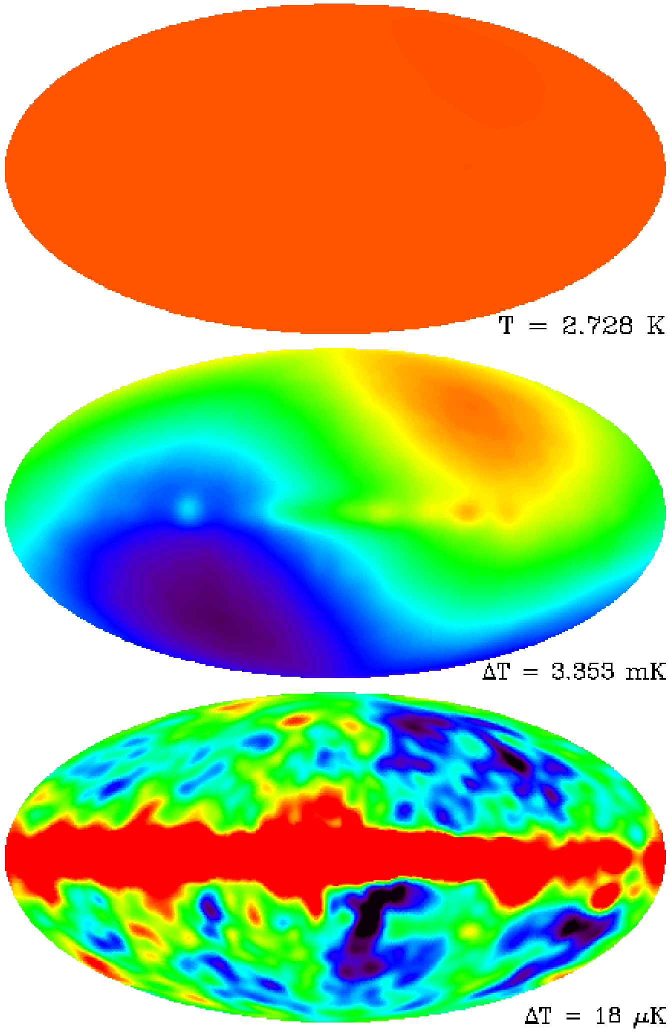
# Linearity

- Intrinsic CMB fluctuations have $\Delta T/T \sim 10^{-5}$

→ First-order (linearised) theory is extremely accurate

→ Each mode evolves independently

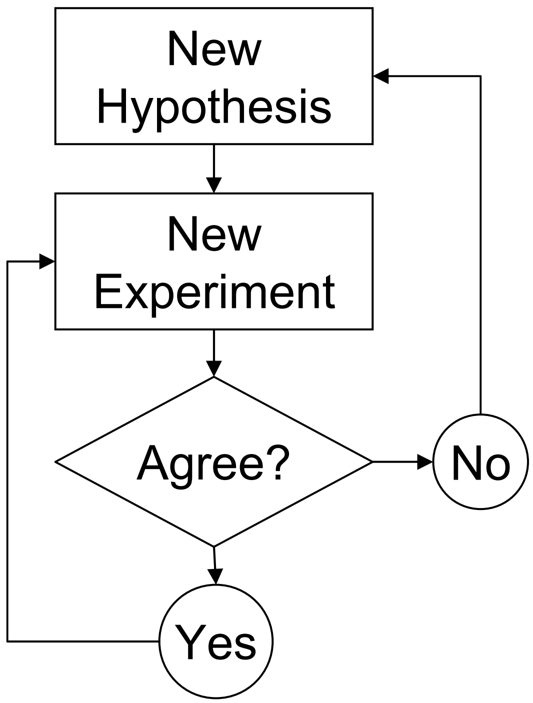
→ Fluctuation spectrum can be predicted in ~ 30 sec on a PC.



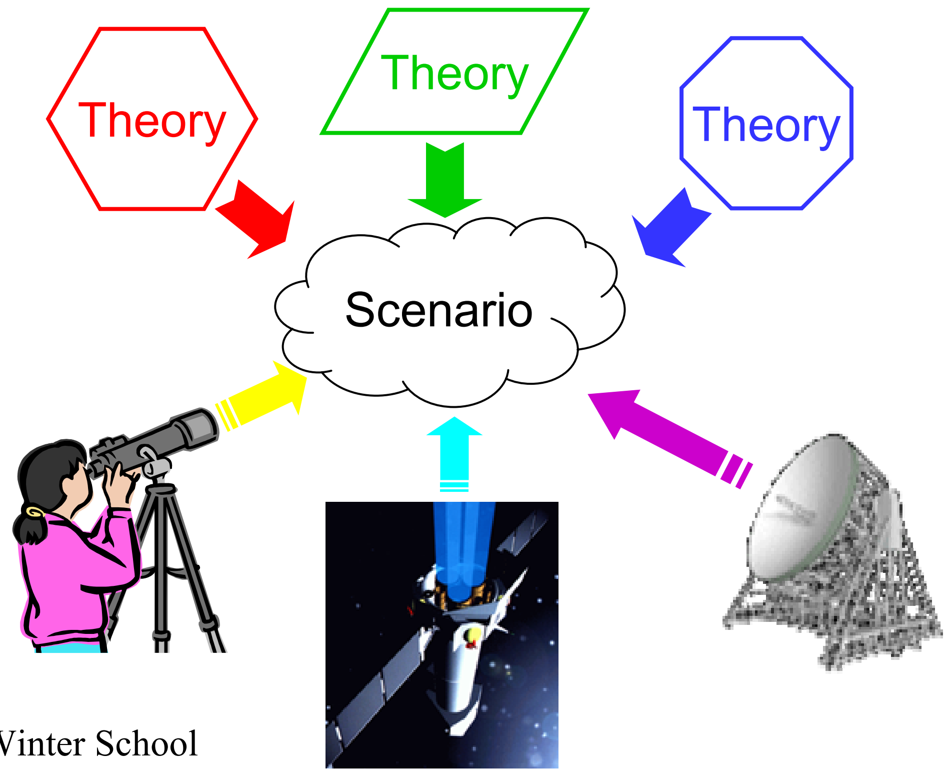

COBE Results

# Scientific method

- School-book model:




- Rees model for Astrophysics:



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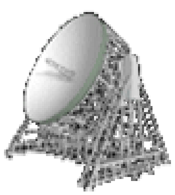
# Theory Ahead

- Cosmic Microwave Background
  - Alpher & Herman 1948, Dicke 1964
- Big-Bang nucleosynthesis:
  - Wagoner, Fowler & Hoyle 1967
- Imprint of inhomogeneities
  - Sachs & Wolfe 1967
- Acoustic fluctuations
  - Sakharov 1965
- Damping
  - Silk 1968, Bond & Szalay 1983
- CMB polarization
  - Rees 1968, Polnarev 1985, Bond & Efstathiou 1987
- Inflation (?)
  - Starobinsky 1980, Sato 1981, Guth 1981, Linde 1982
- Galaxy formation
  - CDM simulations: Peebles 1982 etc



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# Recent Experimental Surprises

- 1989: APM Galaxy survey rules out “Standard Cold Dark Matter”
- 1997: High-z supernova searches rule out open CDM models, imply “dark energy”
- 2003: WMAP discovers early re-ionization
- 2004: non-Gaussianity in WMAP data?

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# Synergy

- Redshifts & Expansion of the Universe:
  - De Sitter, Freidman, Lemaître, Slipher, Hubble
- Dark Energy:
  - Observations (SCP, HZSS, + indirect evidence)
  - Theory: Quintessence (Caldwell, Dave & Steinhardt 1998) etc.
- Details of "concordance model":
  - Observation (WMAP, VSA, ACBAR, CBI etc): meticulous control of systematics; careful measurement of data covariences
  - Theory: (CMBFAST, CAMB etc): exact calculation of grid of cosmologies; Monte-Carlo Maximum likelyhood method.

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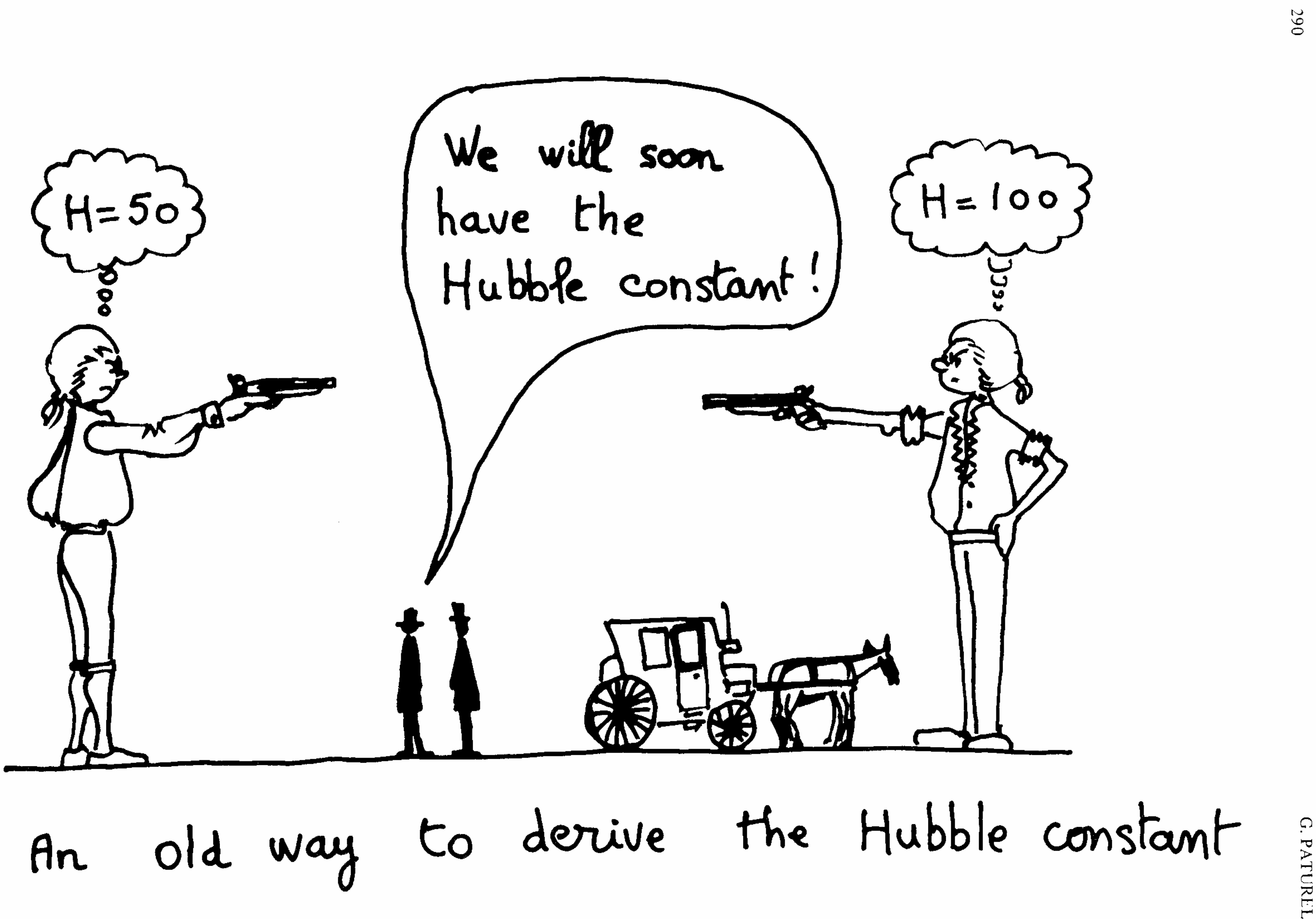


from *Highlights of Astronomy* (1983)

# Precision Cosmology: 2005

- Modern CMB science requires:
  - Errors known to few % accuracy
  - Distribution functions, not just assumption of Gaussianity
  - Full covariance matrices of all potentially correlated parameters
- Error propagation requires linearity in analysis — no CLEAN or MEM
- Bayesian analysis: specify your prior
- Widely different experiments now usually give consistent results: concordance cosmology

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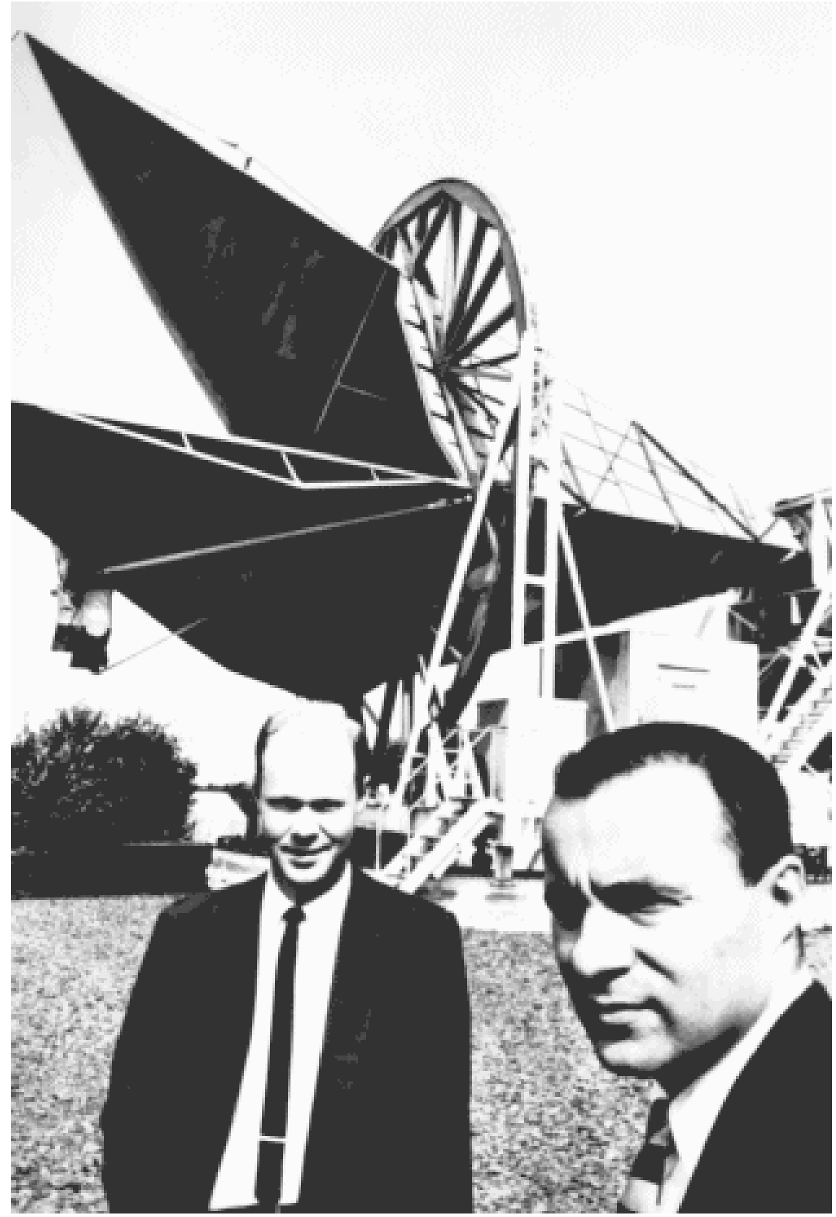
# CMB: Discovery

- 1948: Alpher & Herman predict 5 K radiation, fossil radiation from Big Bang
- 1964: Penzias & Wilson, Bell labs.   Serendipitously discovered it while calibrating their horn antenna
- Princeton team under Dicke unlucky.
- 1979: Nobel prize awarded.



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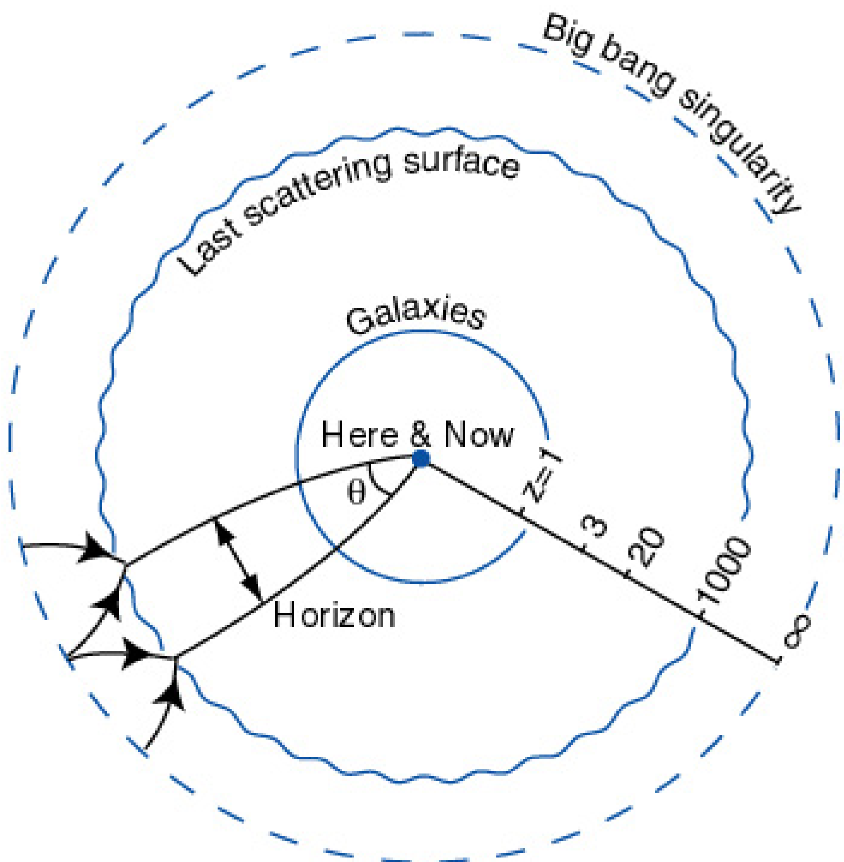
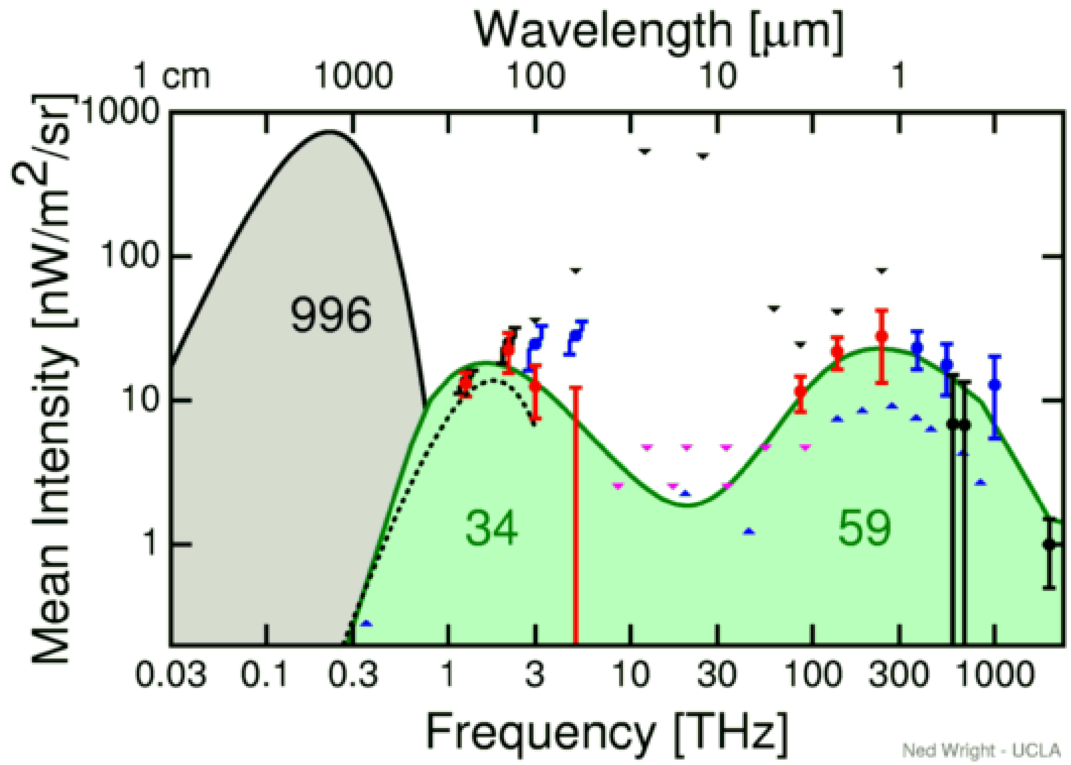
# Problem of Isotropy

- Horizon at last scattering, the "surface" of the CMB, is about 1°. That is, an object at the time of last scattering with a size equal to the particle horizon at that time would subtend ≈1° as seen from here and now.
- Yet CMB is isotropic to 1 part in $10^5$ all over the sky!



# Radiation Density

- Grey: CMB.
- Green: Far IR background: from dust heated by absorbed starlight.
- Blue: Near IR background: redshifted starlight.
- Total CIB: ~8% of CMB.
- NB: data vs. model!



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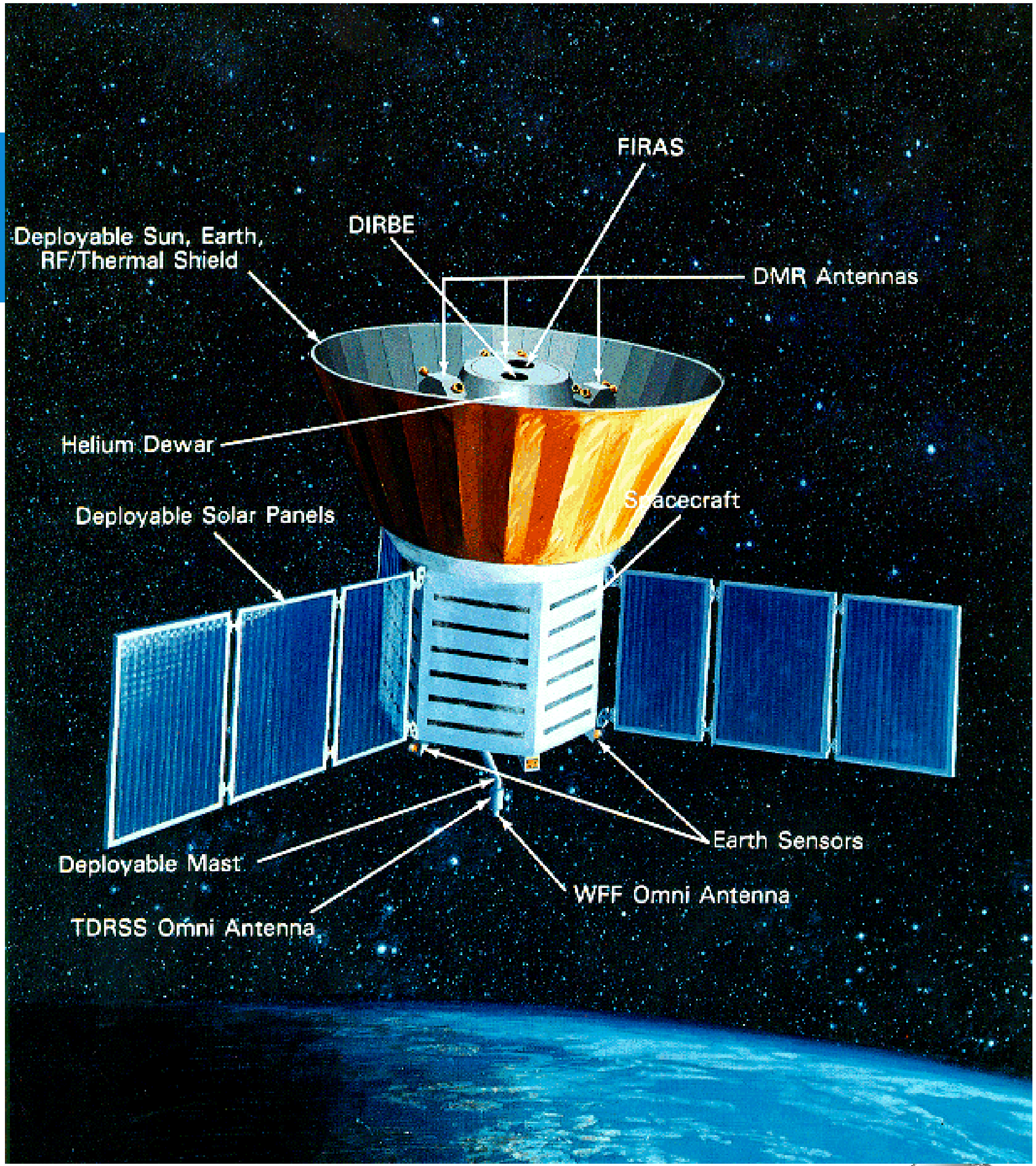
# COBE

- Launch 1989
- Observed to 1994
- FIRAS:
  - CMB is perfect black body,
  - T = 2.725 +/-.002 K
- DMR:
  - Anisotropy detected,
  - ΔT = 18 μK on quadrupole scale
- DIRBE:
  - Cosmic IR Background detected



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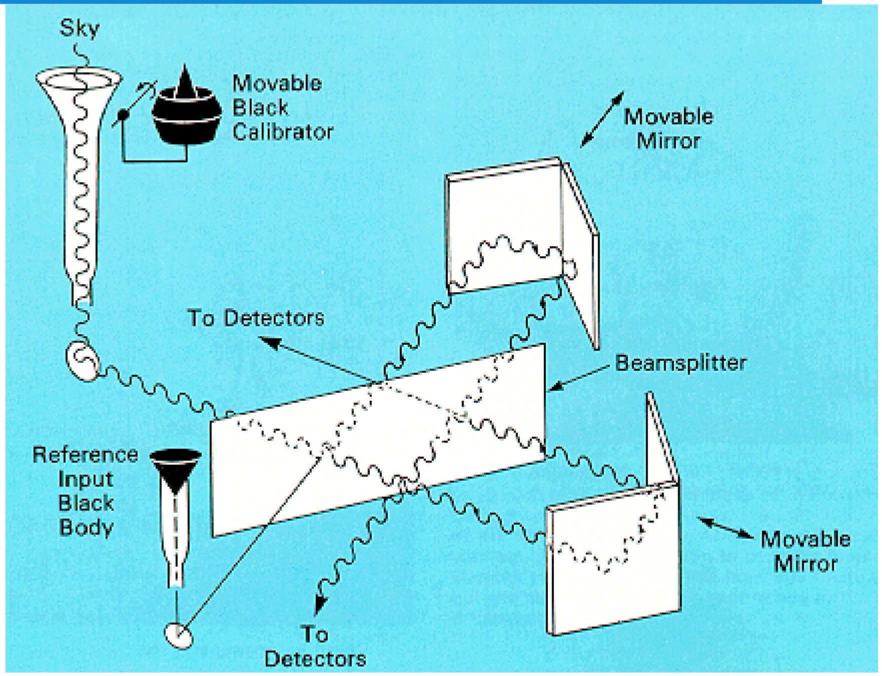
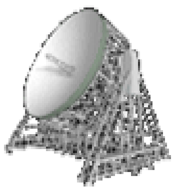
# FIRAS on COBE

- Far Infra-Red Absolute Spectrometer
- Michelson Fourier spectrometer
- Double calibration scheme:
  - Reference load spectrum continuously subtracted from sky
  - Movable 'cold plug' Calibrator on identical signal path to sky corrects for small differential errors in reference




- Overall accuracy limited by measurement of $T_{Cal}$
  - By factor of 10!



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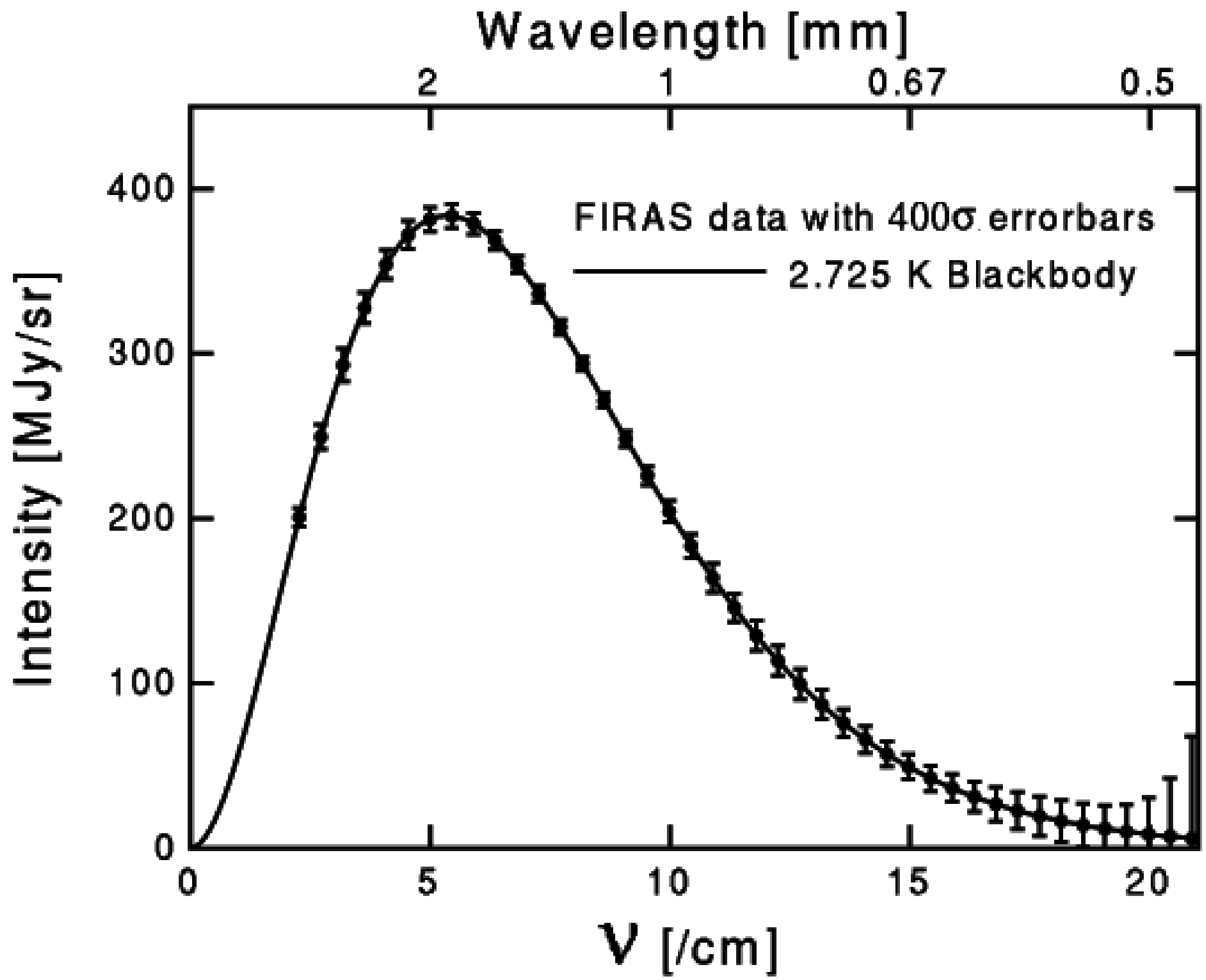
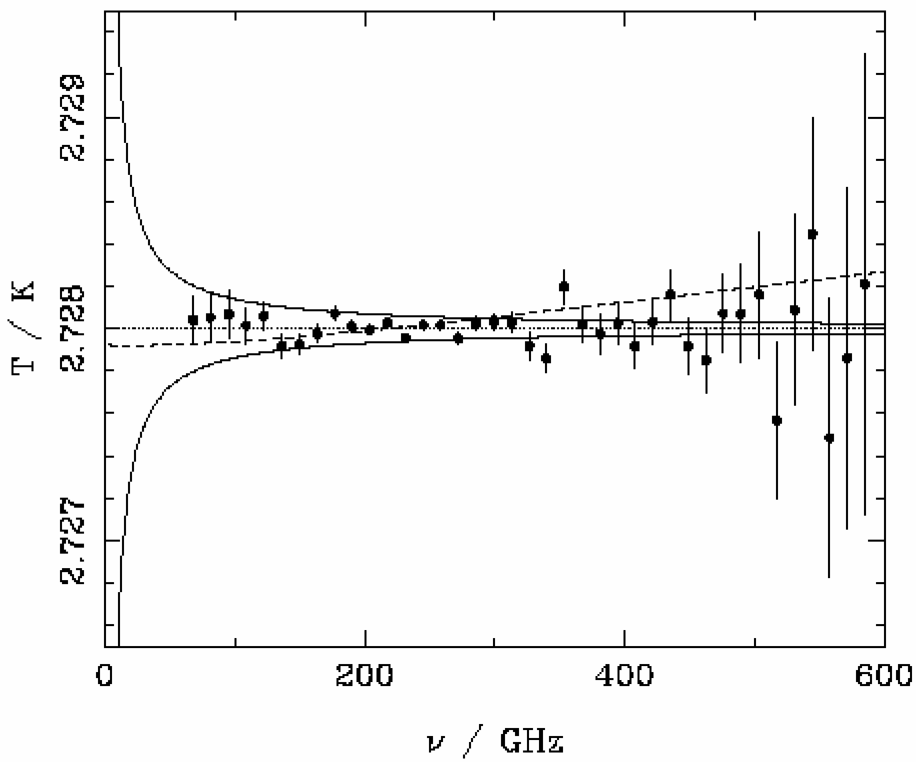
# The Final COBE FIRAS spectrum




400σ!

# COBE Spectrum Residuals




(Re-calibration now gives T = 2.7253±0.0007 K)

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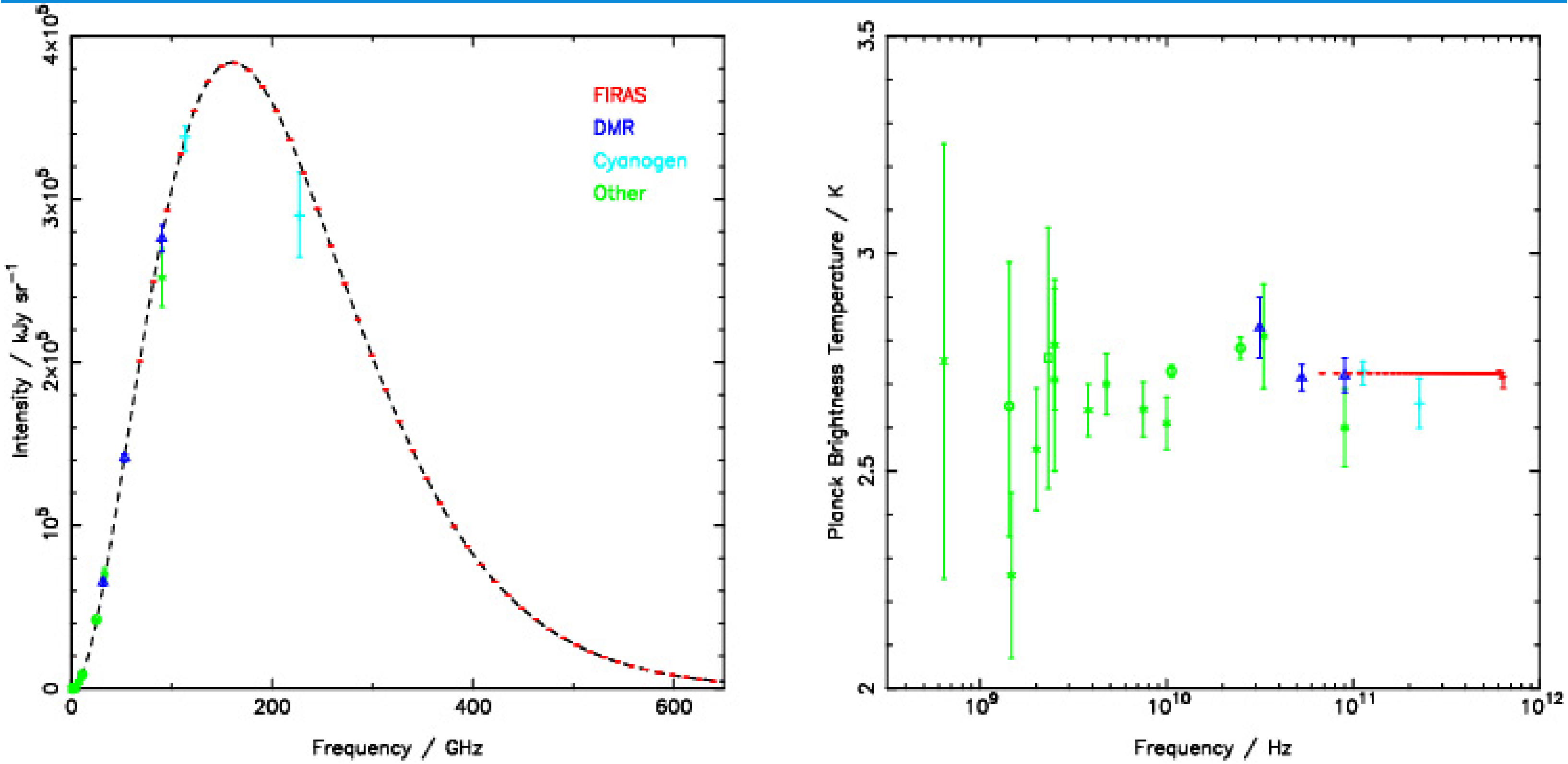
# CMB Spectrum




- Best, direct, evidence for precise thermal equilibrium in the early universe.

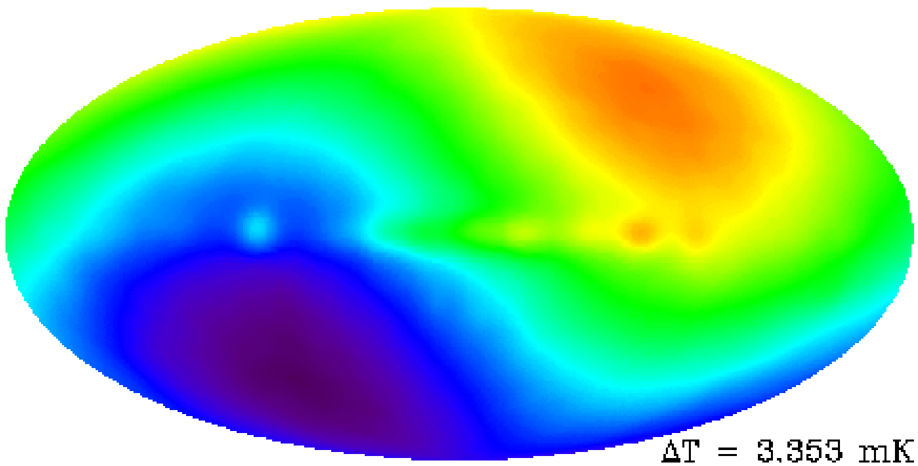
# CMB Dipole

- Wien's law:

$$T\,\lambda_{\max} = const$$

$$\Rightarrow \frac{dT}{T} = -\frac{d\lambda}{\lambda} = \frac{v}{c}$$

  - where $v$ is velocity relative to radiation




COBE DMR

# Problem

Given that

- $I_\nu = (1 - e^{-\tau})\, B_\nu(T)$
- brightness temperature $T_b(\nu) \equiv 2\, k_B\, \nu^2 / I_\nu c^2$

Show that in the Rayleigh-Jeans regime:

$e^{-\tau} = (T - T_b)/T.$

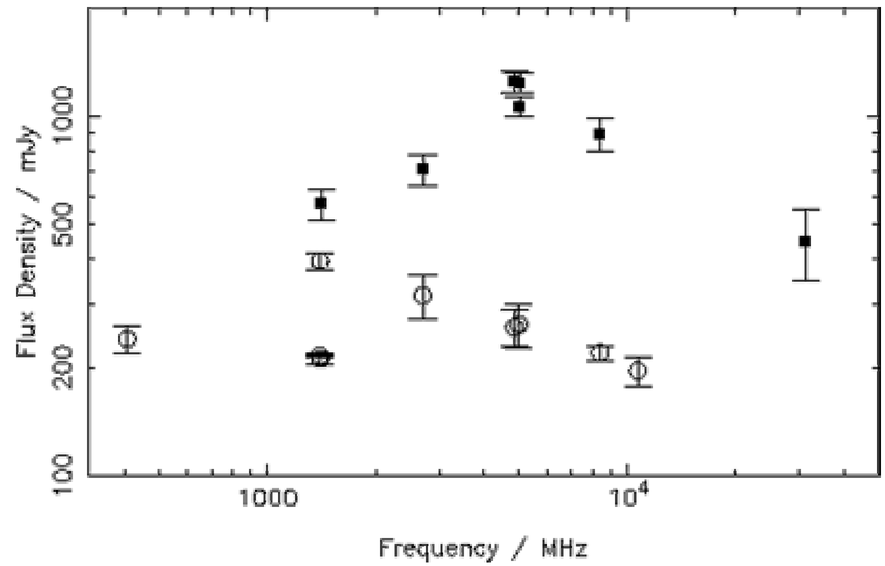
At 10 GHz, the ARCADE experiment found:

$T_b = 2.721 \pm 0.01$ K (Fixsen et al. 2004).

Find the 2σ lower limit on the optical depth given $T_{CMB} = 2.725 \pm 0.002$ K (COBE).

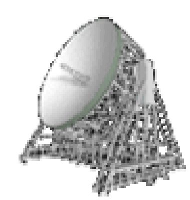
# How Far Away is the CMB?

- Black-body form implies high optical depth in microwave region
  - must be further than any discrete object seen at these frequencies
- SEDs of many extragalactic sources show minimum at ~ 200 GHz…
- But distant blazars have spectra apparently continuous into CMB region
- E.g., small rms uncertainty in observed temperature at 10 GHz implies $\tau > 5$




SEDs of

- PKS 2000–330, $z = 3.773$ (black squares)
- B31428+422 at $z = 4.715$ (open circles).



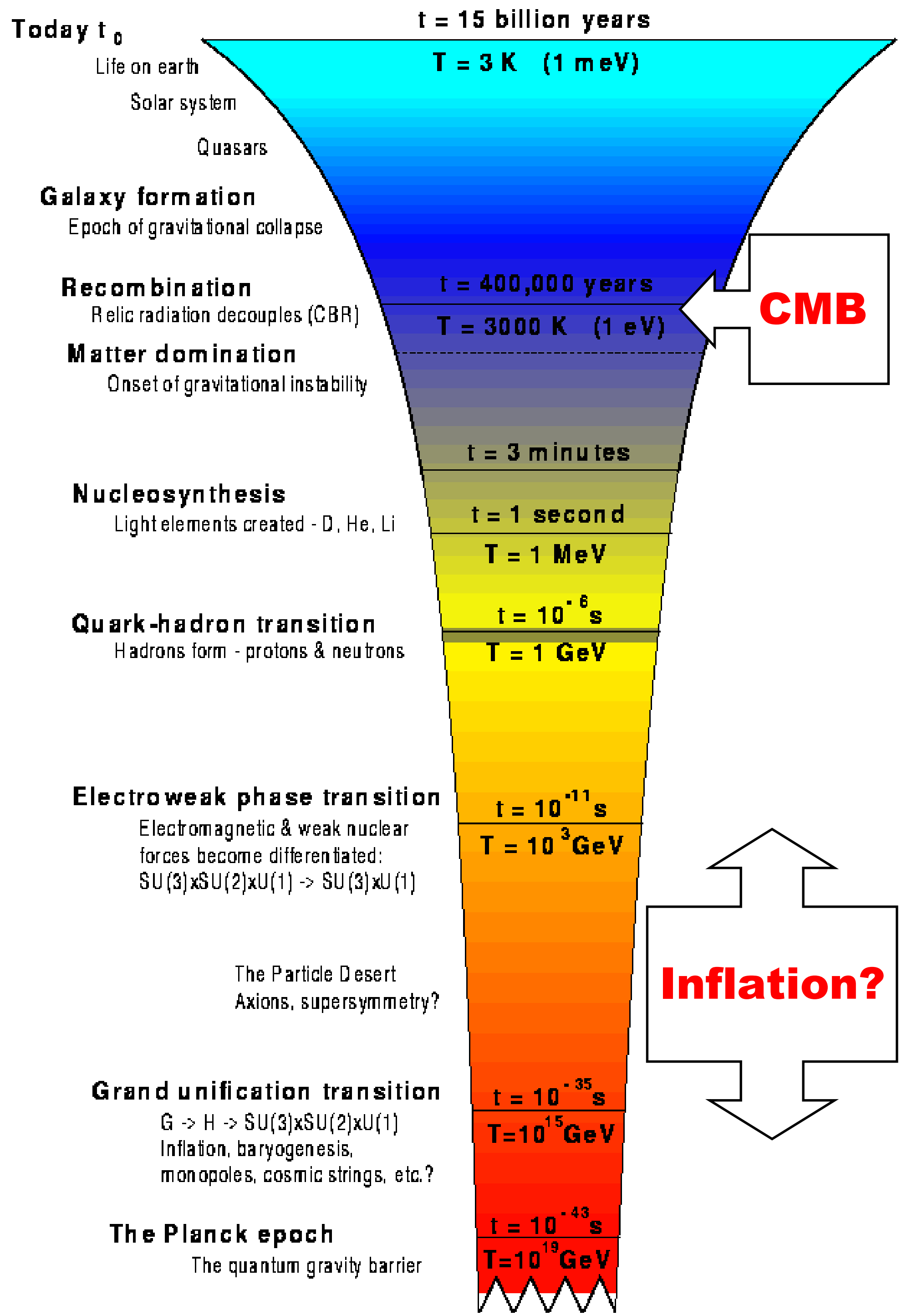
# Origin of CMB Fluctuations

Who really knows? Who can presume to tell it?
Whence was it born? Whence issued this creation?
Even the Gods came after its emergence.
Then who can tell from whence it came to be?

That out of which creation has arisen,
whether it held it firm or it did not,
He who surveys it in the highest heaven,
He surely knows — or maybe He does not!

— *Rig Veda* 10:129
"The Creation hymn"

# Event history



# Origin of CMB Photons

- Short collision time in dense early universe ensures full thermal equilibrium
  - Balance of creation & annihilation
- Gradual annihilation of heavy particles leaves only photons in full thermal equilibrium after 1 sec
- Initially photons rapidly created & destroyed via 3-body interactions with baryons
  - free-free etc
- After few days 3-body interactions too rare, individual photons conserved:
  - Thermal decoupling
- After few years scattering is too low-energy to change photon energy; spectral form preserved
- Path length to scattering remains short for ~ 400,000 yrs until…

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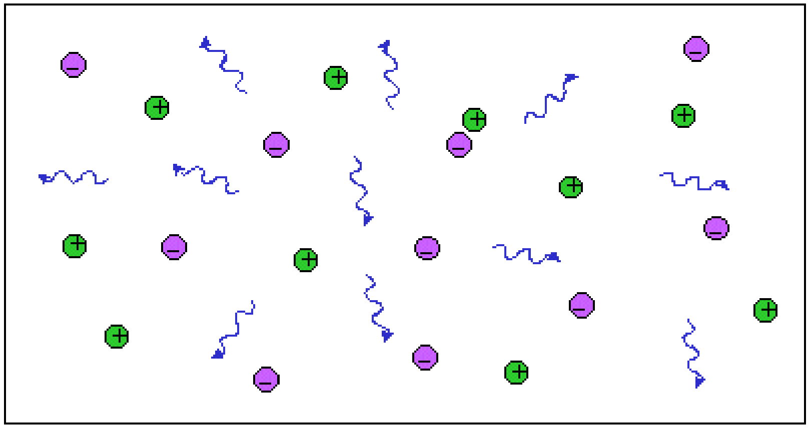
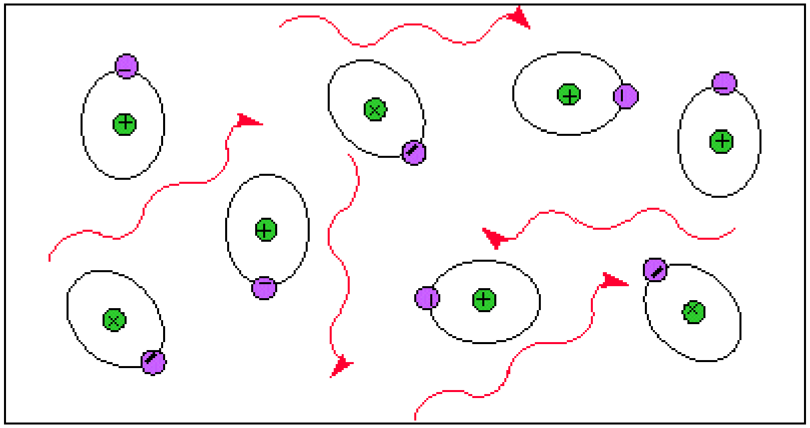
# Recombination

- Before decoupling of matter and energy, photons could not travel far before colliding with particles of matter
- After decoupling, electrons trapped by atomic nuclei and photons liberated to free-stream through universe





# Expanding universe

- Proper distance to some galaxy:

$$d(t) = a(t)\, d(t_0)$$

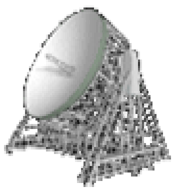
- Present time $t_0$, density $\rho_0$ etc
- Scale factor $a(t) = d(t)/d(t_0) = 1/(1 + z)$
- $a$ is independent of position by isotropy.
- Hubble parameter $H(t) = (1/a)\, da/dt$
- ‘Horizon’, (properly Hubble length): $c/H = 1/H$

# Freidman Equation

- GR predicts for homogeneous, isotropic and flat universe that the Hubble parameter

$$H(t) \equiv \frac{1}{a}\frac{da}{dt} = \pm\sqrt{\frac{8\pi G}{3}\rho(a)}$$

- Solutions:
  - Cold matter $(\langle v\rangle << c,\ \rho \propto a^{-3}):\quad a \propto t^{2/3}$
  - Radiation $(\gamma >> 1,\ \rho \propto a^{-4}):\quad a \propto t^{1/2}$



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  - Radiation $(\gamma >> 1,\ \rho \propto a^{-4})$: $a \propto t^{1/2}$
  - Cosmological constant $(\rho = \Lambda/8\pi G)$: $a \propto \exp\left(\sqrt{\Lambda/3}\,t\right)$

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# Parameters

- Critical density:

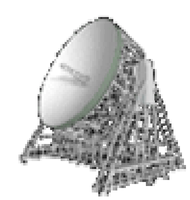
$$\rho_c(t) = \frac{3}{8\pi G} H^2(t)$$

- Density parameter for component X:

$$\Omega_X(t) = \rho_X(t) / \rho_c(t)$$

  - Usually quoted for present day, $t_0$
- Total density parameter $\Omega_o = \rho(t) / \rho_c(t) = \sum_i \Omega_i$

- Cosmological parameters:
  - Present time, $H_0$
    - or h = $H_0$/100 km/s/Mpc
  - Contents: $\{\Omega_i\}$, $T_0$
  - Initial fluctuation power spectrum (Amp, spectral index, etc)
  - Physical parameters including neutrino masses, equations of state of exotic matter and "dark energy"
  - "Gastrophysics" e.g. redshift of first stars



# Key Times in History

- Matter-Radiation Equality:

  $z_{EQ} \approx 1 + z_{EQ} = \Omega_m / \Omega_r \approx 3200$

  $t_{EQ} \approx 60{,}000$ yr

- Last Scattering or photon decoupling:

  $z_{LS} \approx 1090$

  $t_{LS} \approx 400{,}000$ yr

- Re-ionization (just after first stars or QSOs form)

  $z_{reion} \sim 6 - 20$

  $t_{reion} \approx 180 - 1000$ Myr

- Acceleration Starts:

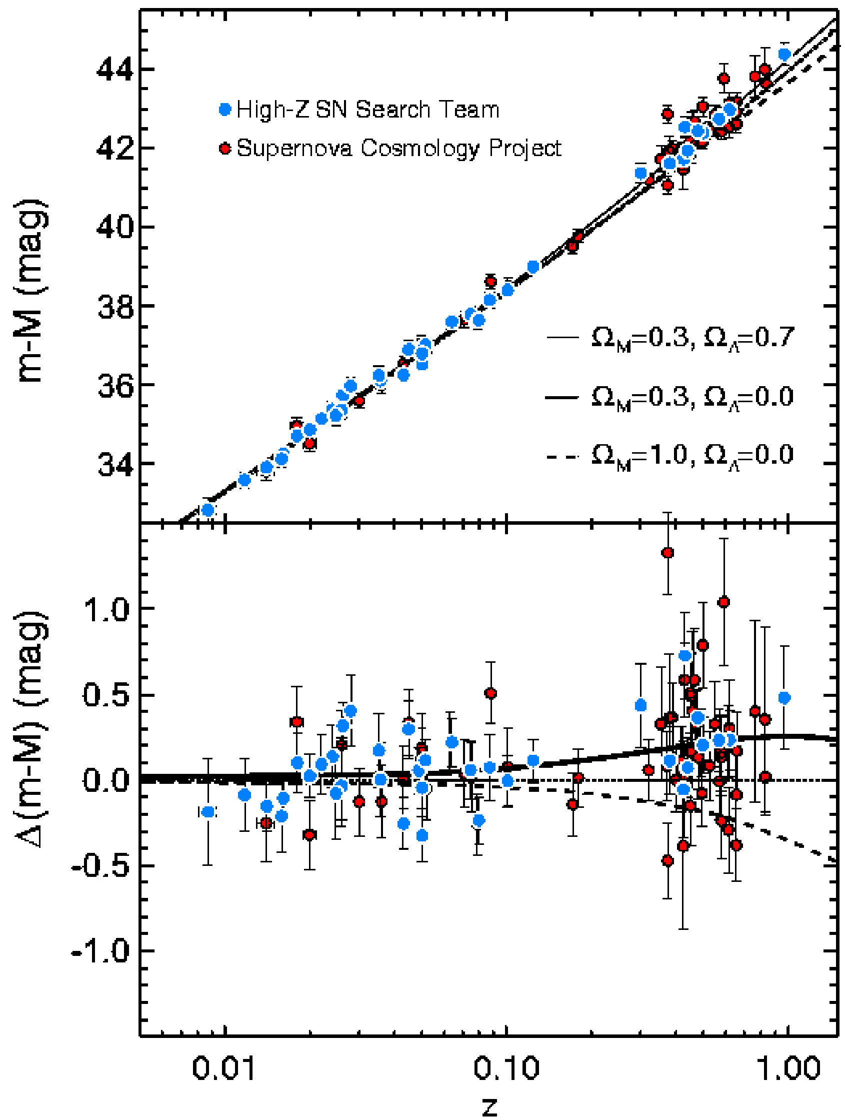
  $\rho + 3P/c^2 = 0 \Rightarrow \Omega_m a^{-3} = 2\Omega_\Lambda$

  $\Rightarrow 1 + z_{ACC} = (2\Omega_\Lambda / \Omega_m)^{1/3} = 1.76$

  $t_{ACC} \approx 7$ Gyr (about half present age of universe).

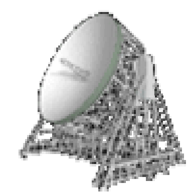
# $\Omega_\Lambda$

- Supernova Ia Hubble diagram:
- If SNIa are good standard candles, at $z \sim 0.7$ they are fainter than expected in a decelerating universe.



# Dark Energy

- No-one expected $\Lambda$!
- Key property is negative pressure, $p = -\rho$, which drives expansion
- Don't jump to conclusion it is this simple — label unknown negative-p component Dark Energy
  - e.g. quintessence, $p = w\rho$ , with 'equation of state' $w < -1/3$



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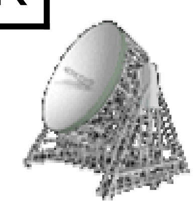
# History of the Universe

| | | |
|---|---|---|
| $10^{-43}$ sec | Planck time | $10^{32}$ K |
| ~$10^{-37}$ sec | Inflation | |
| $10^{-35}$ sec | Fundamental particles, Grand Unification | $10^{28}$ K |
| $10^{-28}$ sec | Defects | ~$10^{20}$ K |
| $10^{-5}$ sec | Neutrons, protons, Electroweak unification | $2 \times 10^{12}$ K |
| 1 sec | $e^-$ / $e^+$ annihilation | $2 \times 10^{10}$ K |
| 500 sec | Fusion, D $\rightarrow$ He | $10^{9}$ K |
| 5 days | Thermal decoupling | $2 \times 10^{7}$ K |
| 60,000 yr | Matter-radiation equality | 9,000 K |
| 380,000 yr | Last scattering | 3000 K |



# The Inflationary Scenario

- Scalar field with non-zero 'zero point energy' → effective cosmological constant.
- Exponential growth during inflationary epoch
- At some later time, zero-point energy converted to ordinary (relativistic) particles: 're-heating'
- Apparent horizon << 'true' horizon

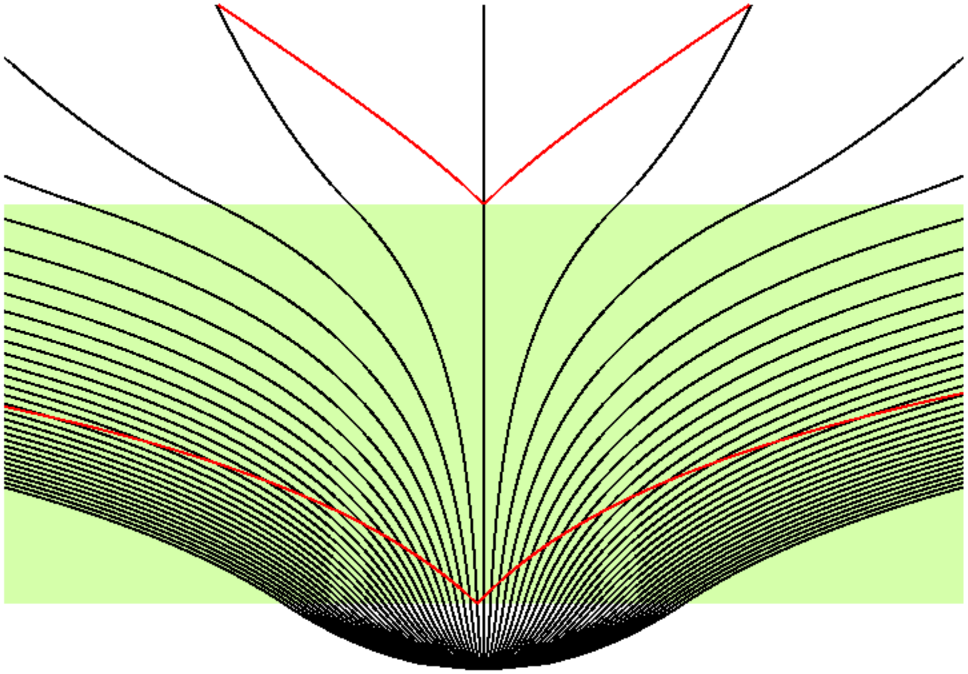


Image: Ned Wright

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- At some later time, zero-point energy converted to ordinary (relativistic) particles: ‘re-heating’
- Apparent horizon << ‘true’ horizon
- Exponential expansion gives $k \to 0$

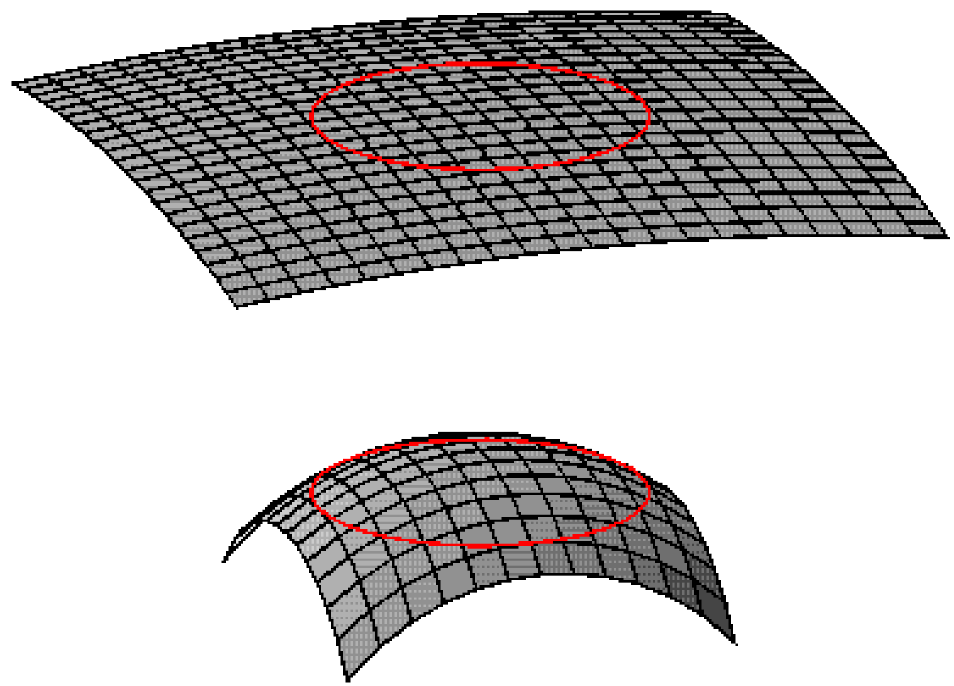

Image: Ned Wright

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# Inflation and Structure Formation

- Quantum virtual particles near event horizon can be separated to give real particles
  - Hawking radiation (black holes)
  - Density fluctuations (inflation)

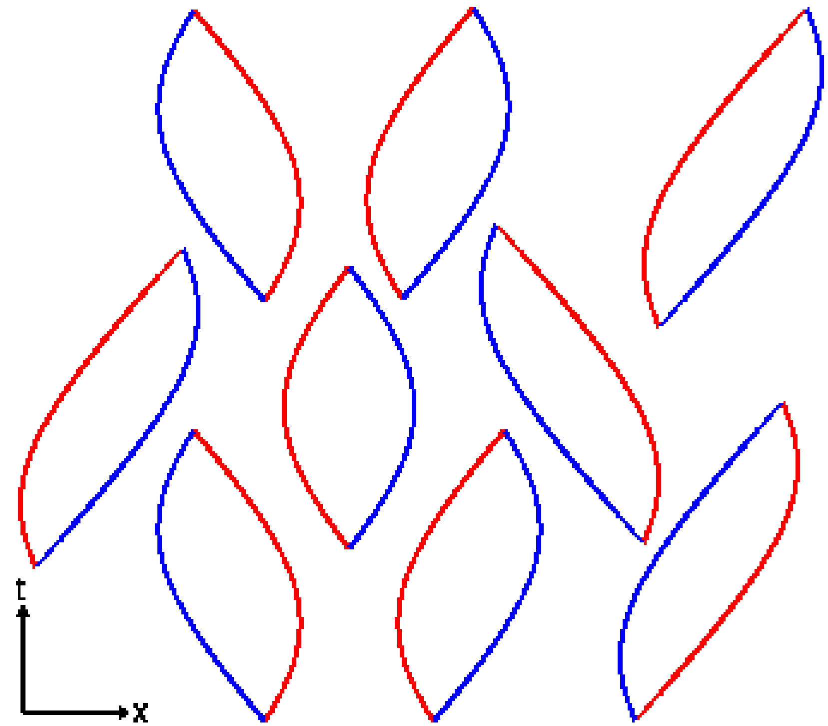
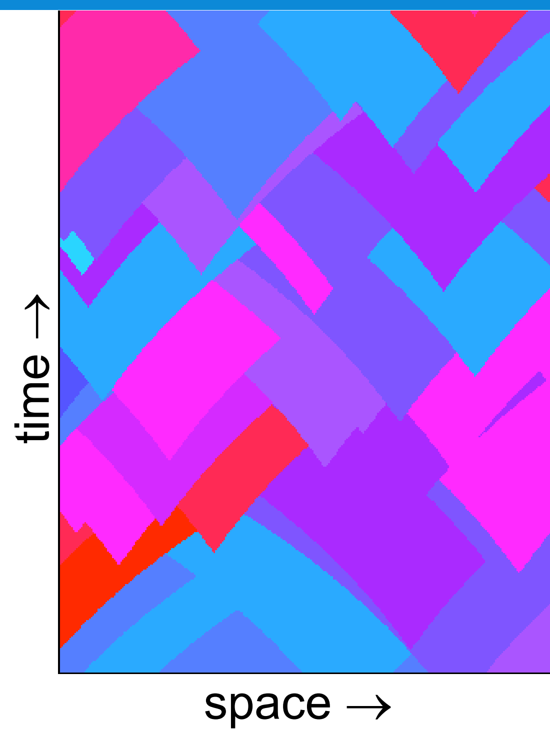
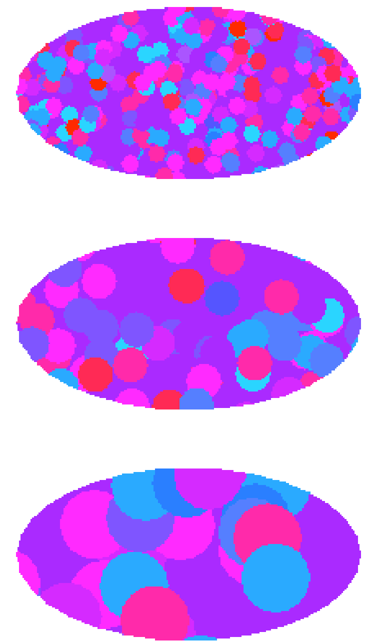



Image: Ned Wright

# Fluctuations

- In inflation, fluctuations are caught up & stretched to macroscopic size by the rapid expansion.
- Remain as very low amplitude fluctuations in density, pressure etc.
- Fluctuations happen continuously → wide range of different length scales, just as observed.
- Exponential expansion → equal time intervals give equal logarithmic range of scales: natural power law



- All structure in the Universe may result from quantum fluctuations during inflation!

# Fourier Modes

- Work in co-moving co-ordinates **x**.
  - Physical coordinate $\mathbf{r} = a\,\mathbf{x}$
- Co-moving wavevector **k**
  - Physical wavelength $\lambda = 2\pi a/k$
- Assume periodic boundary conditions in box with comoving size L → discrete spectrum
- For any function g: $$g(\mathbf{x},t) = \sum_{\mathbf{k}} g_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$
- $g_{\mathbf{k}}$ are complex of course.
  - Hermitian for real $g(\mathbf{x})$

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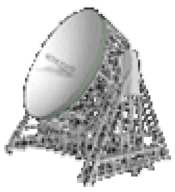
# “Gaussian Fluctuations”

- Inflation predicts that fluctuations are:
  - Uncorrelated in amplitude and phase
  - Phases: uniformly distributed random variables
  - Amplitudes: zero-mean random variables
  - Rms amplitude independent of orientation of **k** (statistical isotropy)
- $\Rightarrow$ real-space fluctuations are
  - Gaussian
  - statistically homogeneous

  in ensemble of universes.
- If fluctuation power spectrum converges at low |***k***|, there is an outer correlation length
  - on scales L much larger than this, Gaussianity and statistical homogeneity apply in one universe (ergodicity)



# Horizon Entry

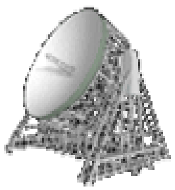
- Roughly, regions more distant than horizon are out of causal contact
  - including peaks & troughs of low-$k$ modes
- During inflation, horizon ~ constant physical length $\Rightarrow$ shrinking in co-moving coords
  - formerly coupled regions can pass out of each others' horizon.
- In decelerating universe, horizon expands faster than $a = 1/(1+z)$, so larger and larger waves 'enter horizon', i.e. peaks & troughs come into causal contact.

# Power Spectra

- By convention, define power spectrum

$$P_g(k) = \left(\frac{L}{2\pi}\right)^3 4\pi k^3 \left\langle \left|g_{\mathbf{k}}\right|^2 \right\rangle$$

  - "power" ($\propto$ amp squared) per unit logarithmic interval in $k$
- Definition assumes statistical isotropy.
- Mass scale associated with wavenumber $k$
  - $M(k) = \rho_0 / k^3$
  - Independent of time for cold matter.



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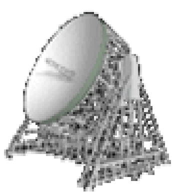
# Harrison -Zel'dovich spectrum

- Power spectrum $P(k) \propto k^{n-1}$
  - By convention!
- $t$ (or $R$) dependence different in different gauges, but for $n$=1, fluctuation power on horizon-entry is independent of $t$, hence 'scale-free'
- Inflation predicts $n$ very close to 1 over a very wide range of $k$. Exact value depends on detailed physics of the inflaton field.
- Edward Harrison (1970) and Yakov Zel'dovich (1972) predicted a scale-free spectrum essentially on observational grounds:
  - Lack of primordial black holes implies $n \leq 1$ on small scales (large $k$)
  - Lack of very-large-scale structure in the universe implies $n \geq 1$ on large scales (small $k$)



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# Fluctuations in what?

- After nucleosynthesis (say $t$ > 1 hour), we have
  - Radiation: photons & neutrinos
  - Matter: CDM & baryonic matter (p, $e^-$, $\alpha^{2+}$)
  - Gravitational waves (in some models).
- Photons & baryons interact via scattering, so form a photon-baryon fluid.
  - In the radiation era, photons dominate the energy density (and so the pressure) in the fluid.
  - $\Rightarrow$ sound speed is relativistic: $c_s = c/\sqrt{3}$.
- Other components can fluctuate independently.
- Notation: relative density fluctuation: $\delta = \Delta\rho/\rho$

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# Types of Fluctuations

- Inflation predicts adiabatic fluctuations:
  - constant entropy/unit mass (≡ photon/matter ratio).
  - all components fluctuate together initially.
- Alternative (orthogonal) mode is isocurvature: fluctuations in matter in anti-phase with fluctuations in radiation.
  - CMB observations rule this out.

# Scalar & Vector modes

- Peculiar (perturbed) velocity: $\mathbf{v}(\mathbf{x},t)$
- Fourier components $\mathbf{v_k} = \mathbf{v_k}^{scalar} + \mathbf{v_k}^{vector}$
  - $v^{scalar}$ parallel to **k**, can be written as $\nabla V$
    - Longitudinal/irrotational modes
  - $V_k^{vector}$ perpendicular to **k**, no potential
    - Transverse/rotational modes
    - Vorticity entirely from vector part.
    - Not generated by vacuum fluctuations
    - Vector modes only decay as universe expands

# Tensor Modes

- Metric perturbations in GR include an additional tensor mode, corresponding to gravitational waves.
- Naturally generated by inflation, with amplitude relative to scalar mode defined by energy-scale of inflation, $E_I$
  - tensor-to-scalar power ratio $r \sim (E_I/10^{17}\mathrm{GeV})^4$
  - cf. GUT scale ~ $10^{15}$-$10^{16}$ GeV
- Rapidly decay on scales much smaller than (present) horizon.

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# Structure Formation

It's nice to know the computer understands the situation, but I would like to understand it too.

— Eugene Wigner

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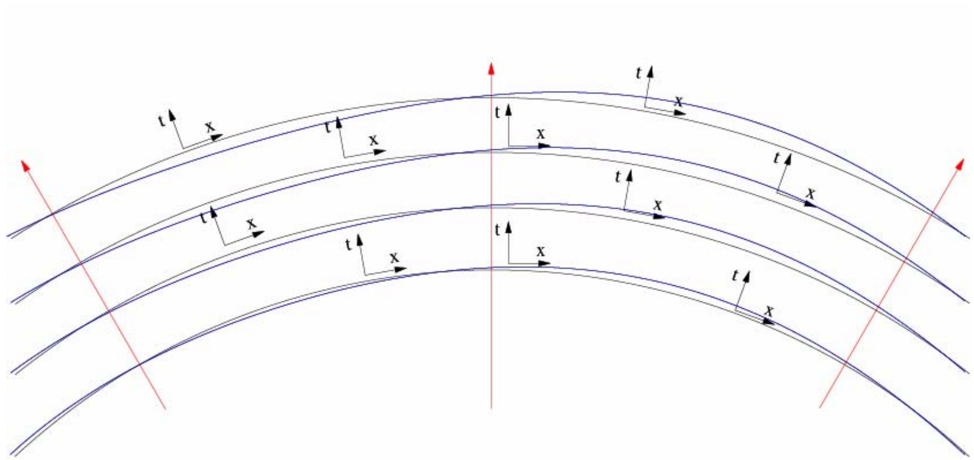
# Perturbations in GR

- Freidman models have preferred coordinate system of 'fundamental observers' who see universe as homogeneous & isotropic
- Perturbations break symmetry:
  - Mathematical description of perturbations depends on chosen coordinate system ('gauge')
  - Major effect on 'outside horizon' relations
  - No effect on observable consequences



# Jeans Length

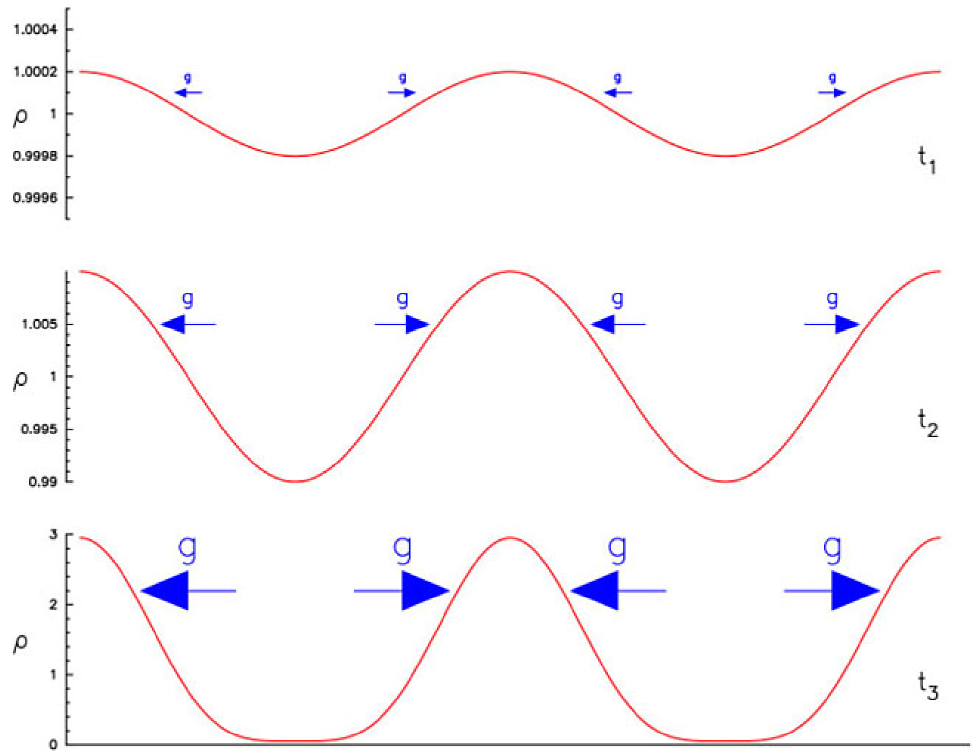
- Newton: Blob with density $\rho$ collapses in $t \sim 1/\sqrt{G\rho}$.
- Jeans: *unless* collapse time > sound crossing time $\sim c_s/L$.
  - Sound waves "tell" interior of blob to increase pressure to oppose collapse.
- Ripples with $\lambda < \lambda_J \equiv c_s\sqrt{(\pi/G\rho)}$ oscillate as standing sound waves,
- If $\lambda > \lambda_J$, peaks gravitationally collapse.
- $\lambda_J$ is the Jeans Length.

# Problem

- Show that if the sound speed is relativistic, $c_s = c/\sqrt{3}$, the Friedman equation implies that in a flat universe
  - $\lambda_J = (\sqrt{8\,\pi/3})\; c/H$
- What does this imply about fluctuations in the early universe?

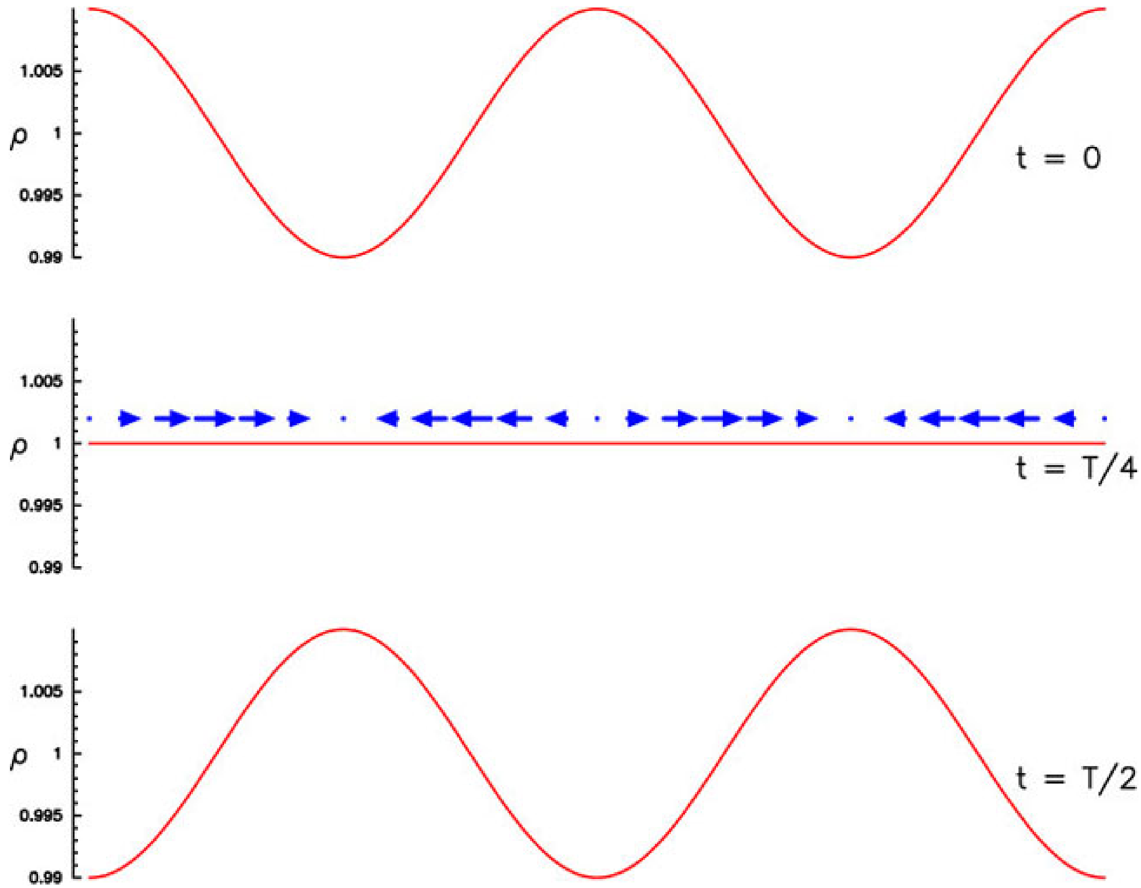
# Structure Formation

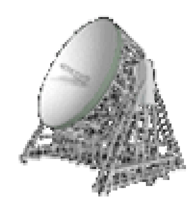
- Density peaks grow in mass by gravity, attract matter from surroundings.
- Growth counteracted by:
  - *Overall expansion of the Universe*
  - *Higher pressure in density peaks.*



# Standing sound wave

- Longitudinal oscillations between density / pressure fluctuations (red line) and velocity fluctuations (blue arrows).
- Frequencies ~picoHertz!



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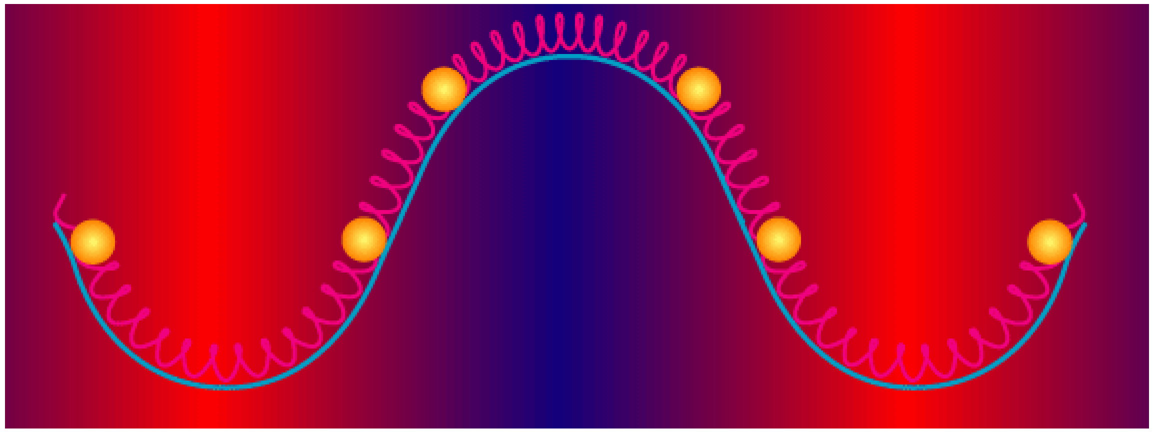
# Acoustic Oscillations

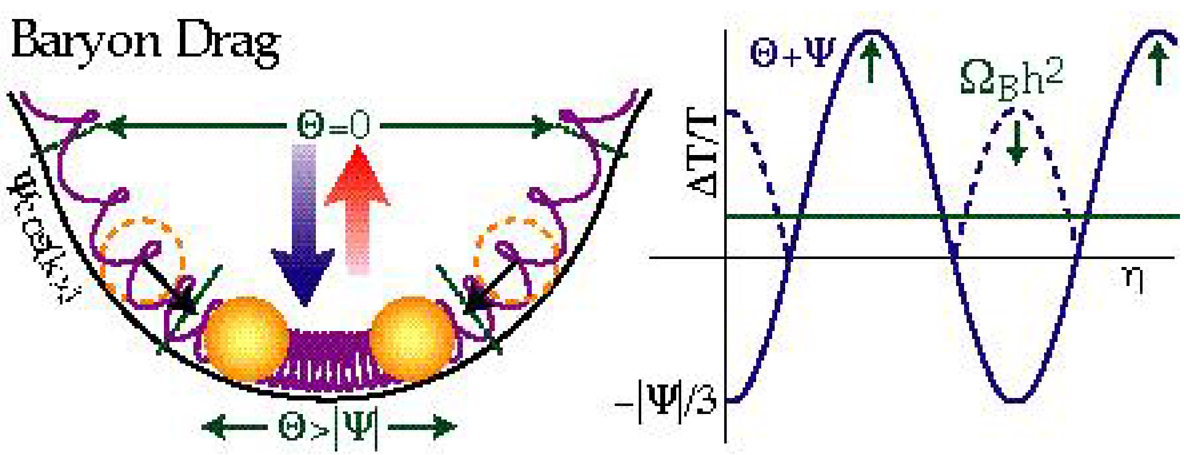


Graphic: Wayne Hu

- Gravitational potential is dominated by CDM: potential depth grows steadily rather than oscillates.
- Baryons → inertia
- photons → pressure (“spring”)
- → Asymmetry between alternate cycles: compressional maximal are alternately in potential wells and on potential hills.

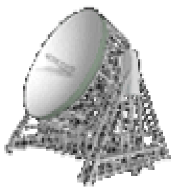
# Baryonic content

- Baryon/photon ratio
  - baryons drag fluid into potential wells
  - More baryons increase height of compressional (odd numbered) peaks, decrease rarefaction (even).



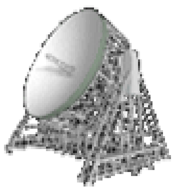
# Synchronicity

- Oscillations start when mode enters horizon
- Depends only on $k$
- Initial oscillation is towards compression in potential wells (gravity drives; pressure responds)
- $\Rightarrow$ all modes with same wavenumber oscillate in temporal phase
  - Different amplitudes
  - Random spatial phase
- Oscillation frequency $\omega = c_s k$
  - $c_s$ = sound speed



# History of Fluctuations

1. λ >> local Hubble radius *c/H(t)* ('horizon'), roughly the distance a photon (better: neutrino) can travel from *end* of inflation until time *t*.
   - Peaks & troughs out of causal contact (since inflation) → wave expands with universe.
2. Horizon catches wave: $k = aH/c$.
   - Inflation predicts (almost) scale-free spectrum: $\delta_k$ on horizon entry same for all $k$, $\delta_k \approx 2\times10^{-5}$ observed (see later).



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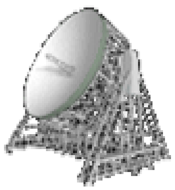
# 3a. Growth in the Radiation era

- Ends at $t_{EQ}$ = time of matter-radiation equality.
- Photon/baryon fluid:
  - $c_s = c/\sqrt{3}$ and $1/\sqrt{G\rho} \sim 1/H$ (from Friedman Eqn)
  - $\Rightarrow \lambda_J \sim c/H$ = horizon scale.
  - $\Rightarrow$ modes entering the horizon during the radiation era behave as sound waves with $\delta_k$ = constant.
  - These waves have co-moving wavelength:

$$\lambda/a \leq ac/H(t_{EQ}) \approx 16\ \mathrm{Mpc}/(\Omega_m h^2) \approx 100\ \mathrm{Mpc}.$$

- CDM: No pressure, but $\delta_k(\mathrm{CDM})$ grows only logarithmically when deceleration is dominated by radiation.



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# 3b. After $t_{EQ}$ and before last scattering ($t_{LS}$)

- Photon/baryon fluid:
  - $\rho_r < \rho_m$, but $\rho_r > \rho_B$, $\Rightarrow c_s$ still ~ $c/\sqrt{3}$,
  - $\lambda_J$ still ~ horizon $\Rightarrow \delta_k$ ~ constant.
  - (modes entering at the end of this period grow slightly).
- CDM: No pressure, theory gives $\delta_k(\mathrm{CDM}) \propto a$

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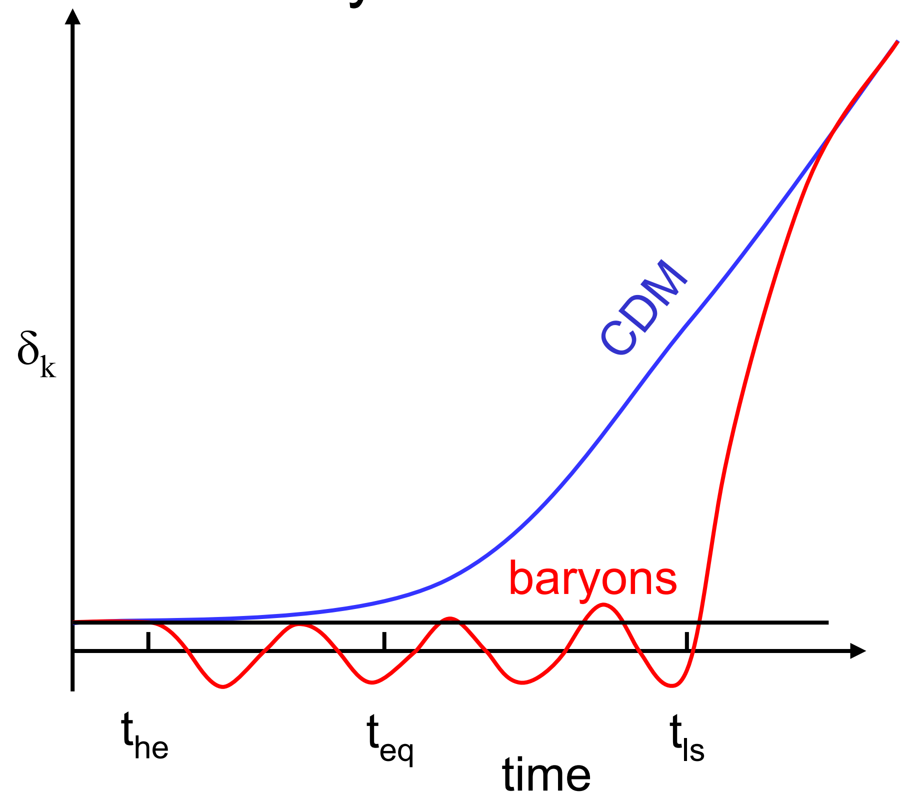
# 3c. linear growth after $t_{LS}$

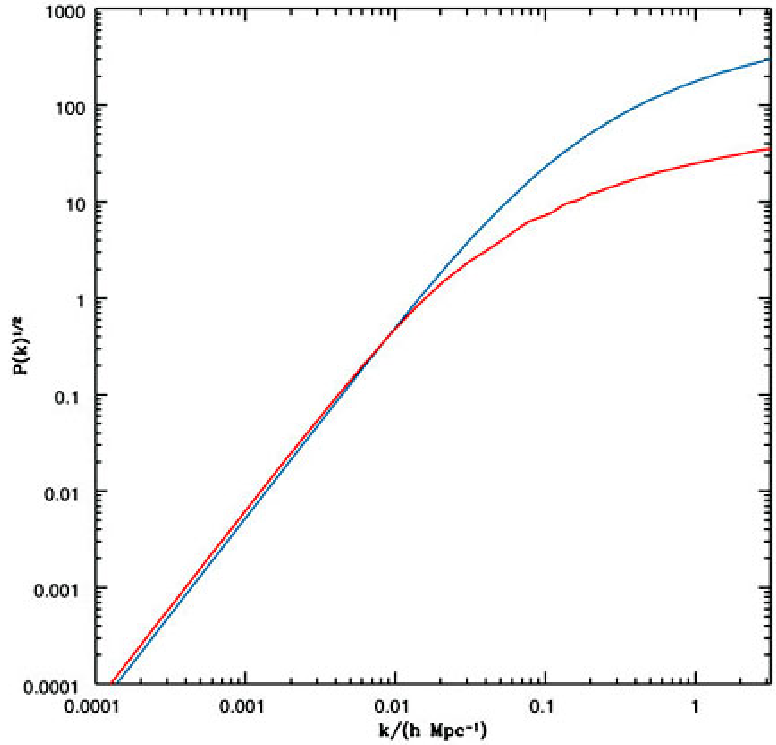
- Photons now decoupled from matter: travel freely in straight lines. Their energy density now negligible.
- CDM: No change, $\delta_k(\mathrm{CDM}) \propto a$
- Baryons: Fall into potential wells made by the CDM.

- Summary:



# Effects of linear growth

- Modes which entered the horizon before $t_{EQ}$, so $\lambda/a$ << 100 Mpc, all have comparable amplitudes $\delta_k$.
- Longer $\lambda$ have progressively lower $\delta_k$, as they enter horizon later, so have less time to grow.




Blue: concordance model.
Red: CDM only ($\Omega_m = 1$)

# 4. Non-linear Growth

- When $\delta \sim 1$: run-away collapse, modes interact.
  - Earliest stars form at $z < 20$, $t > 200$ Myr
  - Galaxies form at $z \sim 10$.
  - Most distant measured galaxy $z = 6.56$.
  - Clusters form at $z \sim 1$.
- Can only be modelled via numerical experiments: computer simulations following up to $10^9$ particles (representing CDM) and also gas flow.

# 5. Gravitational Freeze-out

- When $\Omega_m$ falls below 1, growth of structure halts.
- This is happening now.
- Waves with $\lambda$ > 30 Mpc now back to expanding with the universe, still in the linear phase.
- Can be compared directly with analytical, not numerical, theory.

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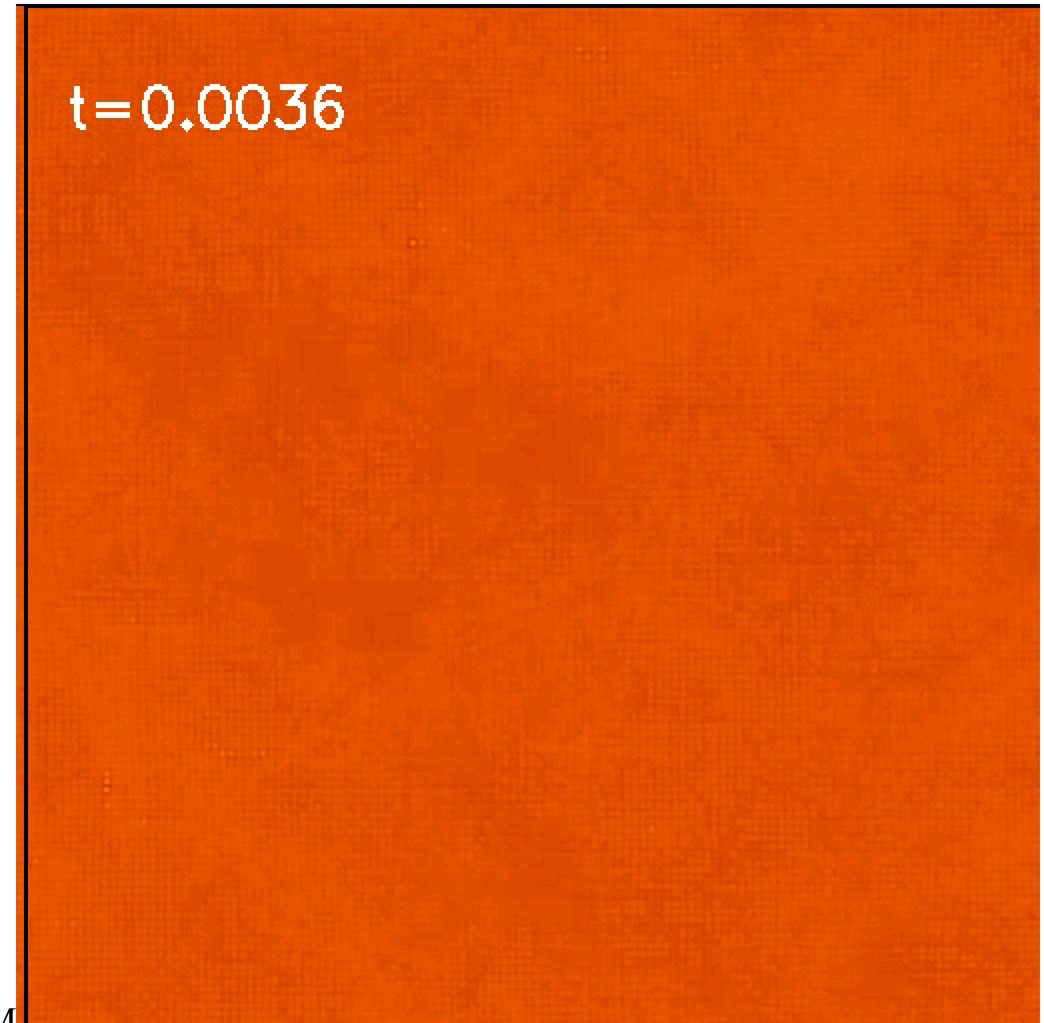
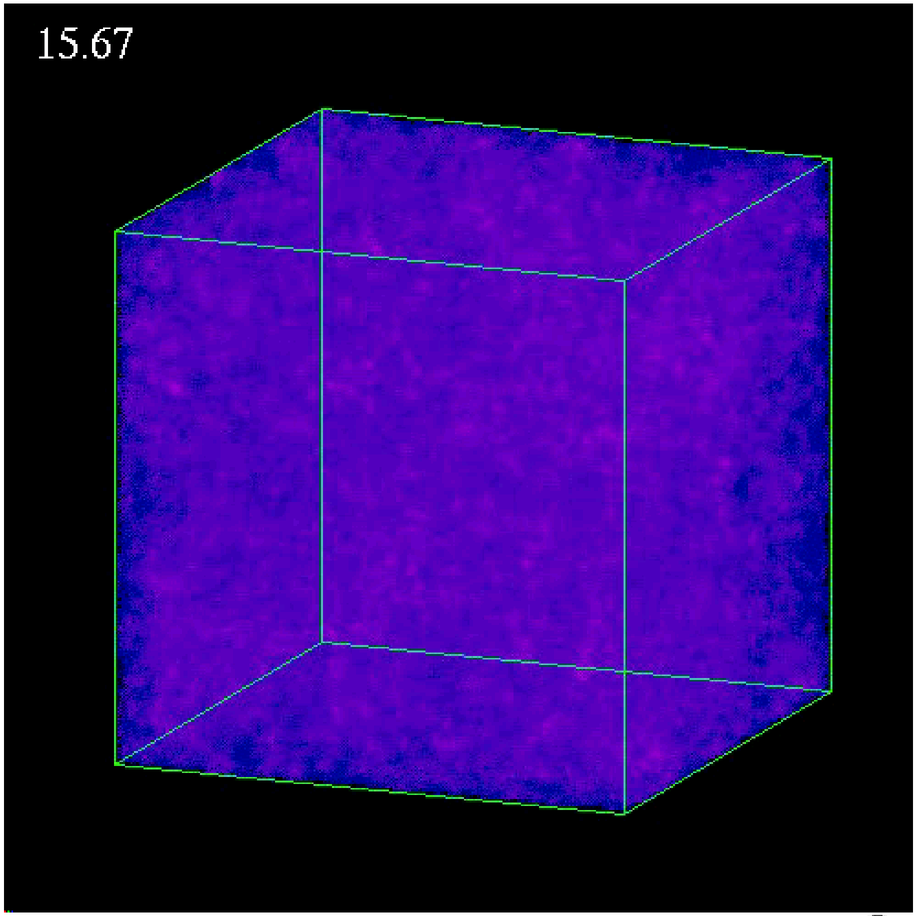
# Simulated Cosmic Web

- 50 Mpc co-moving cube.
- ( Hubble expansion corrected away).
- From VIRGO consortium simulations.



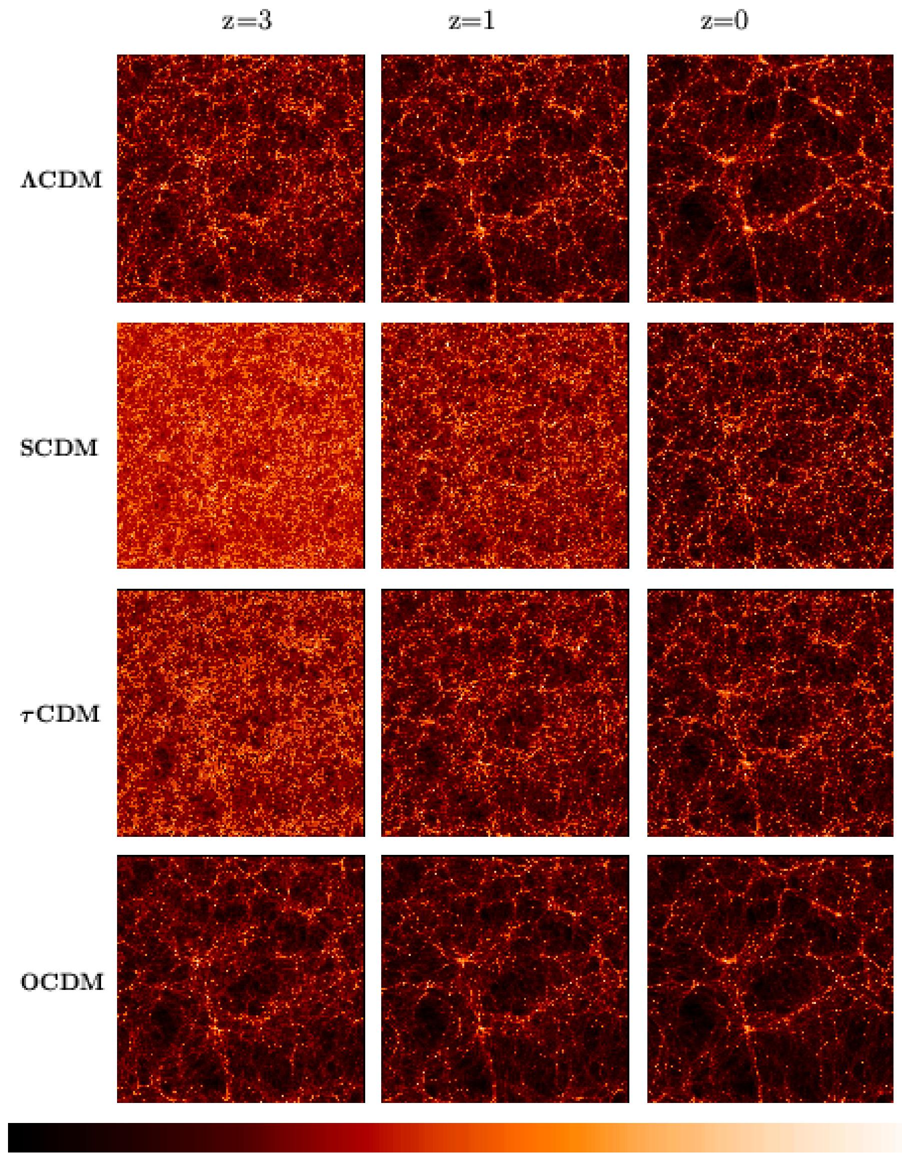
# Simulated Cosmic Web

- 200 Mpc co-moving cube.
- (i.e. Hubble expansion corrected away).
- From VIRGO consortium simulations.



# N-body Results

- Different cosmological parameters give different rates of structure formation
  - Standard Cold Dark Matter ($\Omega = 1$, no dark energy) ruled out as structure forms too late.

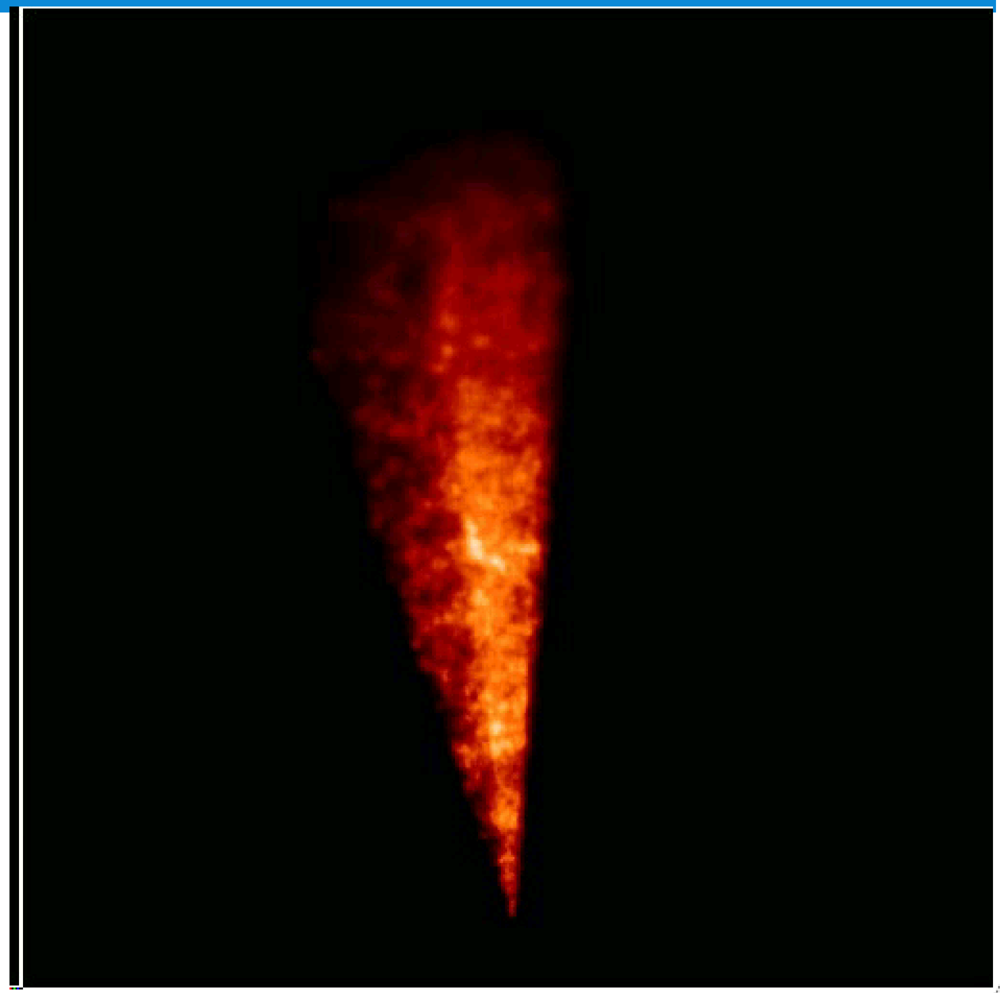



The VIRGO Collaboration 1996

# Observed Cosmic Web

- Results from 2dF galaxy redshift survey.
- Typical distance from Earth:

  ~ 300 Mpc.
- For comparison with theory!



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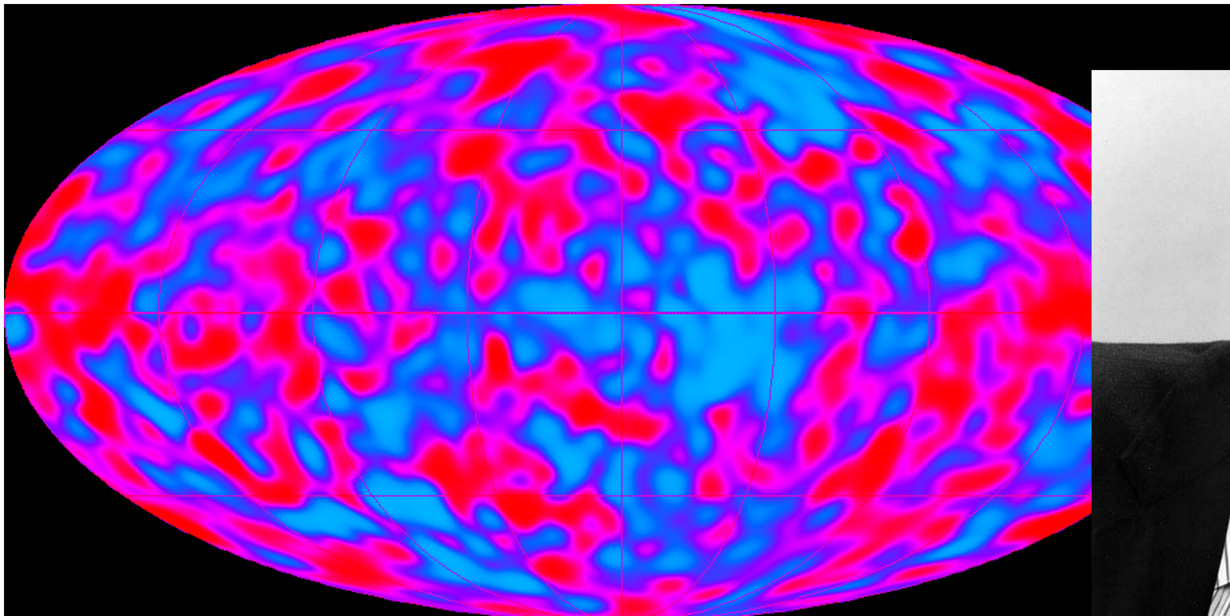
# Fluctuations in the CMB



If you're religious, it's like looking at God.

— George Smoot

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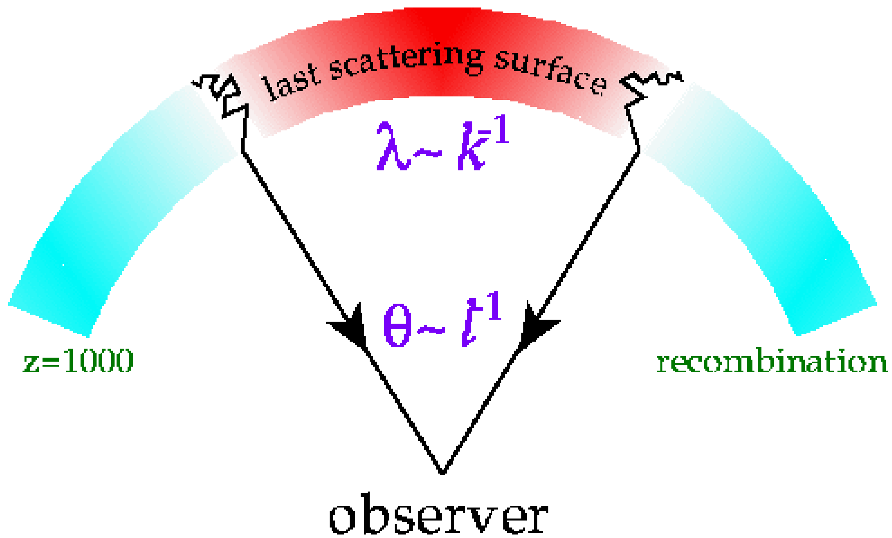
# Recombination & Last Scattering

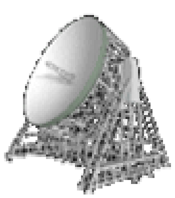
- At 400,000 yrs, *T* falls enough for $p + e^- \rightarrow H$: recombination (really just combination!).
- Process initially described by same physics as stellar atmospheres: Saha equation.
- But in homogeneous universe Lyα can't escape… slows down recombination
  - 2-photon emission
  - Escape via line wings




- Last scattering requires almost complete recomb: $z_{ls}$ = 1088±2 (T = 2970 K)
- $\Delta z_{ls}$ = 194±2 (FWHM)
  - From WMAP.

# Problem

- Estimate the thickness of the last scattering shell (Silk scale):
- Mean time between collisions
  - $t_c \sim (n_e \sigma_T)^{-1}$
  - Assume Silk scale is distance travelled in Hubble time 1/H at $z_{ls}$ by a photon (random walk)
- To get $H(t_{ls})$, recall $\Omega_m(t) \propto \rho_m(t) H^2(t)$
  - What is $\Omega_m(t_{ls})$?
- Data:
  - $\Omega_m = 0.135\ h^{-2}$; $\Omega_b = 0.0224\ h^{-2}$; $\sigma_T = 6.65\times10^{-25}\ cm^{-2}$



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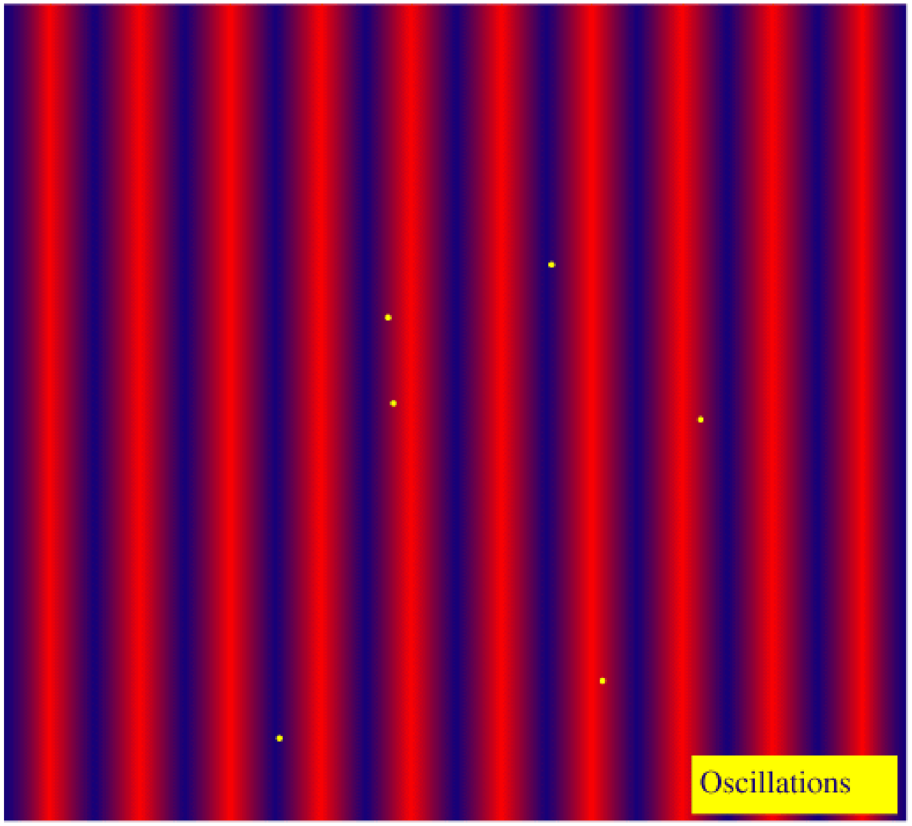
# Fluctuations in the CMB

- CMB photons travel direct to us from spherical surface of last scattering at time $t_{LS}$.
- We *see* sound waves/fluctuations in the photon-baryon fluid at that time.
- NB: sound horizon ~ $c_s t$ ~ $ct/\sqrt{3}$ corresponds to ≈ 0.5° on the sky.




Graphic: Wayne Hu

# CMB analysis formalism

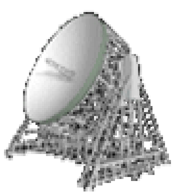
- If $T_0$ is the mean CMB temperature, fluctuations are

$$\Delta T(\theta,\phi) = T(\theta,\phi) - T_0$$

- This can be expressed as a sum over spherical harmonics:

$$\Delta T(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} a_{lm} Y_{lm}(\theta,\phi)$$

- The $a_{lm}$ are multipole moments
  - $\ell = 0$ mean temperature
  - $\ell = 1$ dipole anisotropy
  - $\ell = 2$ quadrupole moment
- WARNING: Theorists frequency expand $\Delta T/T_0$ instead. (Check units!)



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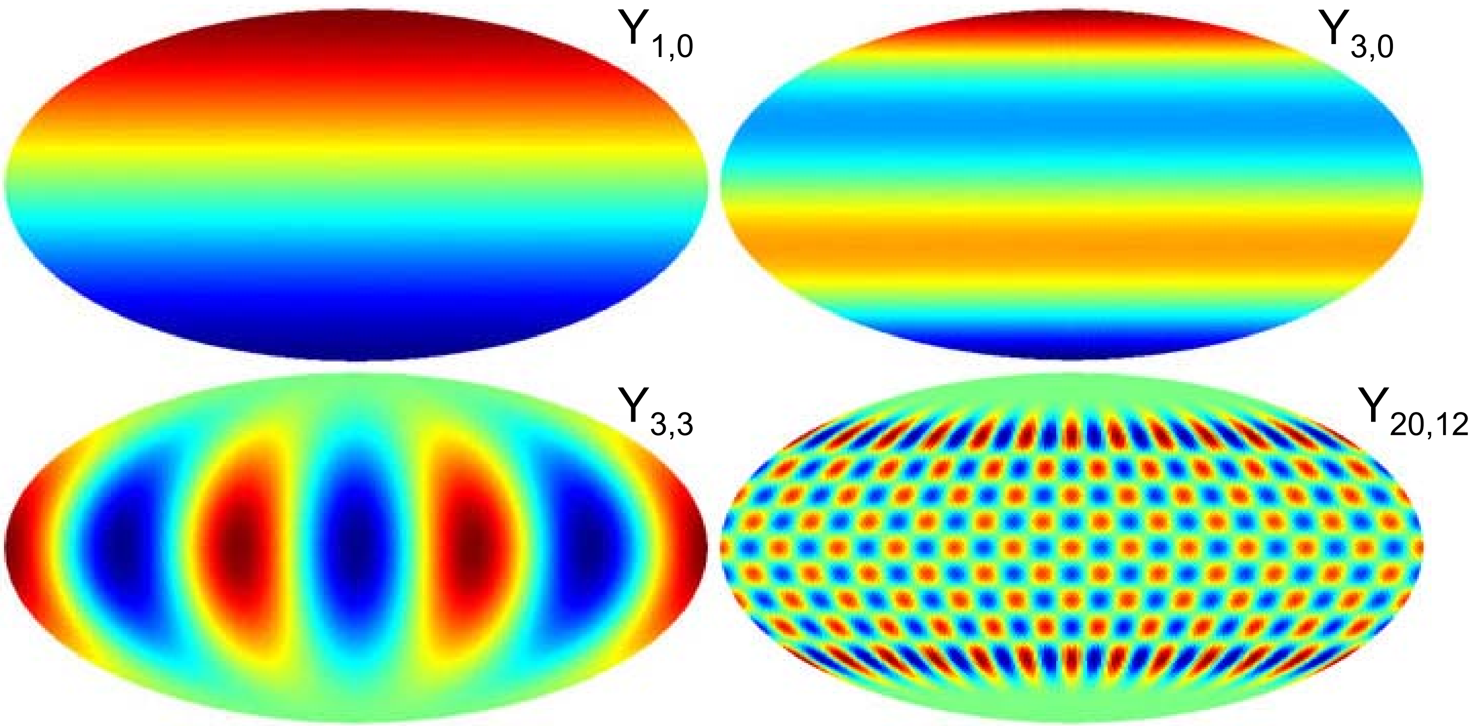
# Spherical Harmonics




- Complete set of orthonormal functions
- Real ("cos") component shown here

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# Spherical vs Fourier Harmonics

- $\ell$ inversely related to angular scale: mode with projected wavelength $\theta$ composed of $Y_{lm}$ with $\ell \approx 2\pi/\theta$
- At high $\ell$, over small regions, relation is almost exact.
- Superposition of *m* modes required for one plane wave, & vice versa.

# $C_\ell$s

- The angular power spectrum of the temperature fluctuations averaged over universes is

$$C_l \equiv \langle |a_{lm}|^2 \rangle$$

- By convention, we plot $\ell(\ell+1)C_\ell/2\pi$ vs $\ell$
  - constant at low $\ell$ for a scale-free spectrum
  - ≈ σ(T) on maps with FWHM ~ 180°/ $\ell$
- If the fluctuations obey Gaussian statistics,
  - each $a_{lm}$ is independent
    - apart from reality constraint $a_{lm} = a_{l-m}{}^*$
  - power spectrum provides a complete statistical description of the temperature anisotropies.

# Primary Fluctuations

Gravity

Doppler

Density fluctuations

Sachs –Wolfe effect

Damping

*Washes out structure <0.1 deg (l~1000)*

Defects

(Strings, textures)

*And tensor models (gravitational waves)*

# Components of ΔT/T

Dipole anisotropy, where $V_S$ is our velocity with respect to the radiation

$$\frac{\Delta T}{T} = \frac{V_S}{c}$$

Gravitational potential or Sachs-Wolfe fluctuations

$$\frac{\Delta T}{T} = -\delta\phi$$

If density perturbations are adiabatic

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{\delta\rho}{\rho} = -\frac{1}{3}\delta\phi$$

*In decreasing order of importance* ↓

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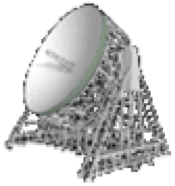
# 3 Regimes for CMB Fluctuations

1. Sachs-Wolfe effect: $\theta > 0.5°$
   - Ripples outside horizon
   - From GR, rms amplitude = $-\delta_k/3$
   - High density $\rightarrow$ cool spots: photons must climb out of potential wells.
2. Acoustic Peaks: $0.5° \geq \theta \geq 0.1°$
   - Standing sound waves in photon-baryon fluid.
   - Waves of same $\lambda$ all in phase!
   - $\rightarrow$ amplitude is an oscillating function of $\lambda$ or $\theta$, representing peaks vs. nulls of the sound waves.



# Problem

- The *COBE* fluctuations corresponded to $T$ = 18 μK. Show that this leads to a value for the density fluctuation on horizon entry of $2\times10^{-5}$.

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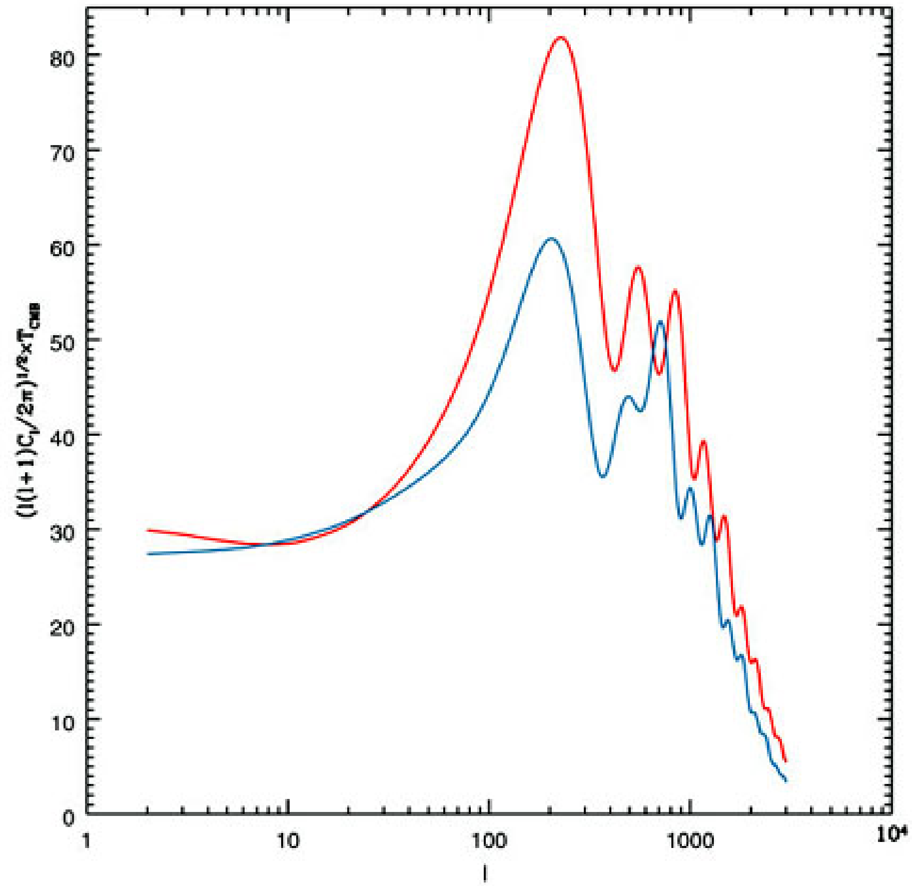
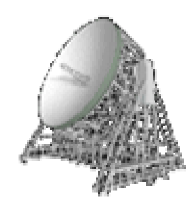
# 3 Regimes for CMB Fluctuations

3. Damping tail: $\theta \leq 0.1°$
   - Finite thickness of last scattering surface ($\Delta z \approx 190$) damps small-scale fluctuations because peaks & troughs seen superimposed.
- Figure:
  - Red: CDM only
  - Blue: Concordance



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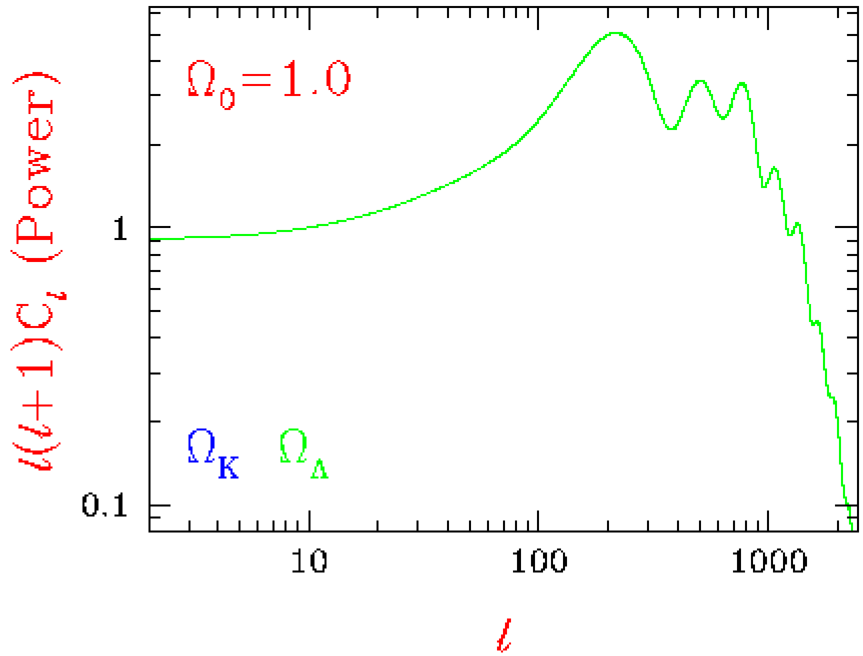
# Why multiple peaks in $C_\ell$ ?

- Acoustic oscillations at given *k* correspond to fixed $\ell$ on surface of last scattering
- Waves with fixed *k* are synchronised: same time of horizon entry, same frequency.
- Phase at $t_{ls}$ varies with *k* (or $\ell$):
  - Higher $\ell$ (shorter wavelength) entered horizon earlier and have gone through more oscillations
  - First peak: modes reaching their first maximum overdensity in potential wells after horizon entry
  - Second peak: modes rebounding to underdensities for first time
  - Third peak: overdensities for second time
  - etc

# What the Peaks tell us

- First peak angular scale is set by sound horizon at last scattering.
- Direct measure of angular size distance to $z$ = 1100
- Sensitive to combination of combination of curvature of space and $H_0$
  - higher ℓ (smaller angular scales) corresponds to negative curvature, lower to positive




- Blue: varies "$\Omega_k$" = 1-$\Omega_0$
- Green: varies $\Omega_\Lambda$ with $\Omega_0$ = 1

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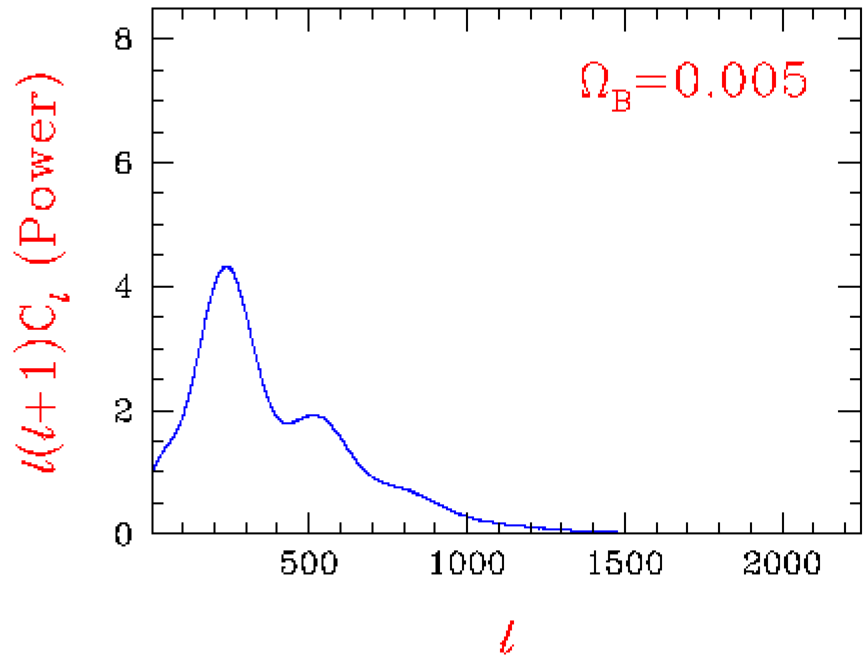
# What the Peaks tell us

- Multiple Peaks imply acoustic nature of fluctuations - argues strongly against defects
- Ratio of 2nd to 1st peak height fixes baryon density



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# What the Peaks tell us

- 3rd peak sensitive to ratio of matter (CDM + baryons) to radiation
- Scale of damping tail set by thickness of last scattering surface (mean free path of photon at last scattering)
  - Provides independent checks on parameters derived from peaks.