# Understanding Radio Interferometry

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# RADIO NOISE AND RADIO WAVES

## RADIO SOURCE EMISSION

### RECEIVED NOISE POWER

UNIT OF FLUX DENSITY, $S$ jansky Jy

$= 10^{-26}\ W\ Hz^{-1}\ m^{-2}$

($\equiv$ optical magnitude)

### SURFACE BRIGHTNESS (of extended source)

FLUX DENSITY / UNIT SURFACE AREA $b$

UNIT Jy / beam area (in radio maps)

PHYSICAL UNIT Brightness temperature $T_b$

The temperature at which a black body would give the same radio emission per unit area

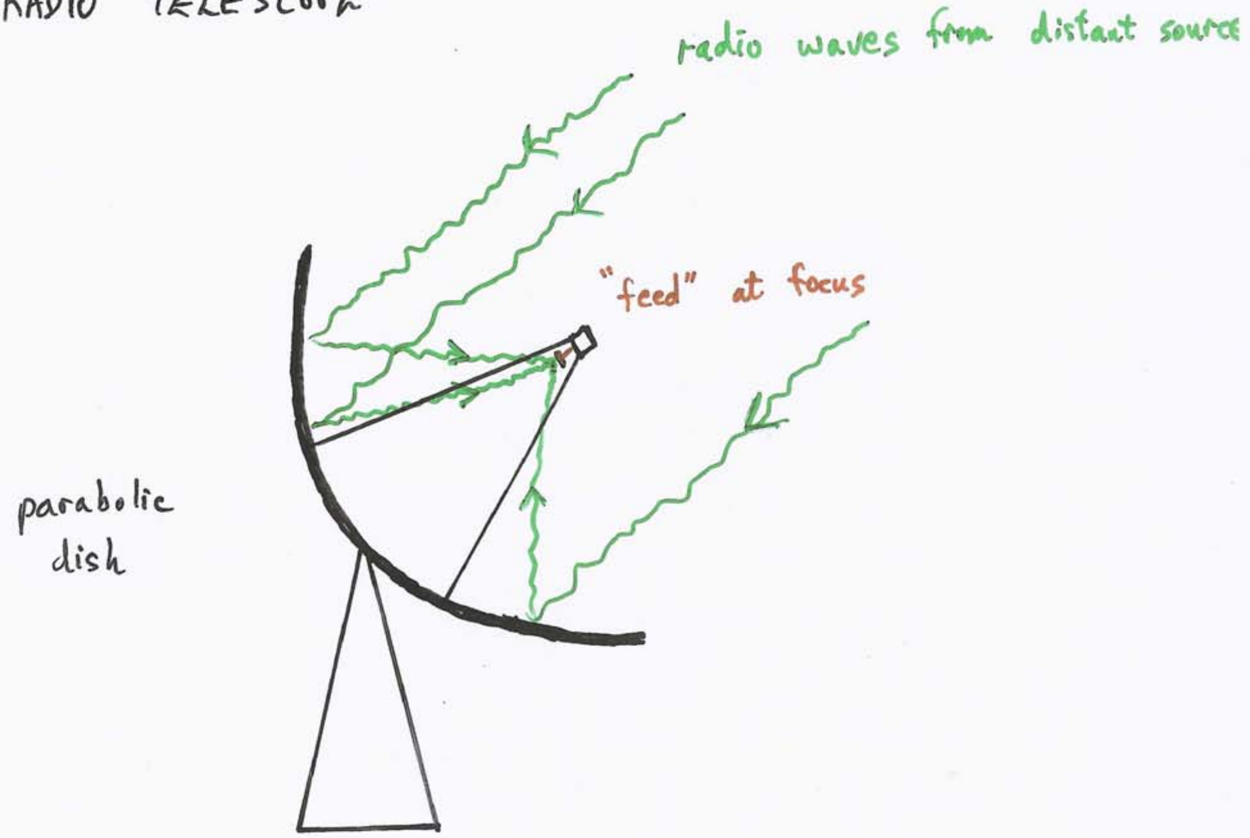
$$T_b = \frac{S \lambda^2}{2 k \Omega}$$

$k$ = Boltzmann's constant
$\Omega$ = observed solid angle

SURFACE BRIGHTNESS IS INTRINSIC (DISTANCE INDEPENDANT)

# MEASURING RADIO SOURCE EMISSION

RADIO TELESCOPE



## DESCRIPTION IN TERMS OF NOISE POWER


FEED PRODUCES NOISE VOLTAGE RERESENTING NOISE POWER FROM RADIO SOURCE

COMPARE WITH JOHNSON NOISE FROM RESISTOR OF TEMPERATURE $T$ ($= kT$ per unit bandwidth)

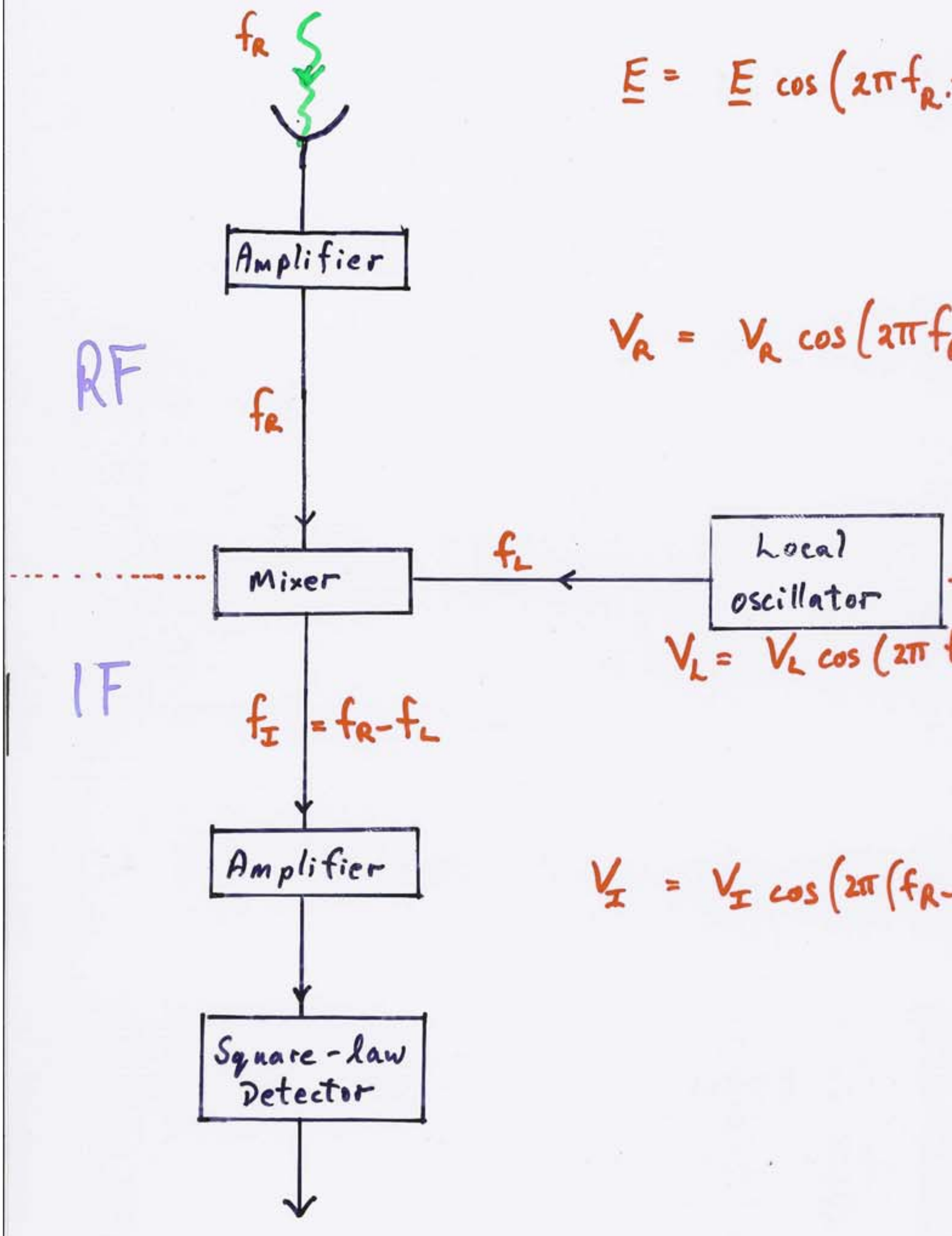
$T_{ant}$ from source $\qquad T_{ant} = g . S$

$T_{rec}$ from receiver

$T_{sys}$ = total system temperature $= T_{rec} + T_{ant}$

$S = g^{-1} \left( T_{sys}(\text{on-source}) - T_{sys}(\text{off-source}) \right)$

## DESCRIPTION IN TERMS OF VOLTAGES




$$\underline{E} = \underline{E} \cos(2\pi f_R . t + \phi_R)$$

$$V_R = V_R \cos(2\pi f_R . t + \phi_R)$$

$$V_L = V_L \cos(2\pi f_L . t + \phi_L)$$

$$V_I = V_I \cos(2\pi (f_R - f_L) . t + \phi_R + \phi_L)$$

$$\langle V_I^2 \rangle \propto T_{sys}$$

## THE FEED (e.g. a dipole)

THE E FIELD IS A 2-D VECTOR
BUT $V_R$ IS A SCALAR

$\Rightarrow$ THE FEED ONLY SAMPLES ONE COMPONENT OF THE E FIELD (A LINEAR OR CIRCULAR POLARIZATION MODE)

## THE RF SIGNAL BAND

$f_R \Rightarrow f_R \pm \frac{b}{2}$

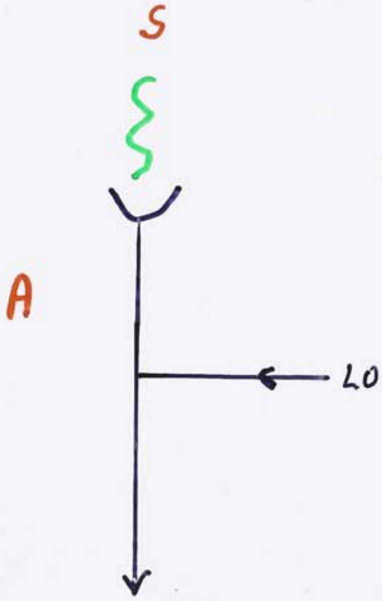
$b =$ bandwidth

- NOISE VOLTAGE IS THE SUM OF SINUSOIDAL OSCILLATIONS AT ALL FREQUENCIES WITHIN THE BAND $b$, EACH WITH ARBITRARY RELATIVE PHASE
- FOR A TIME SHORTER THAN $b^{-1}$ THE VOLTAGE BEHAVES LIKE A PURE SINGLE FREQUENCY AT $f_R$
- AFTER A TIME LONGER THAN $b^{-1}$ THE SIGNAL PHASE HAS CHANGED ARBITRARILY (DUE TO RELATIVE ROTATION ACROSS THE BAND OF THE INDIVIDUAL COMPONENT FREQUENCIES)

$$\langle V_A V_B \rangle = 0$$

# THE IF SIGNAL BAND $f_I \pm b$

THE NOISE CHARACTERISTICS OF THE IF SIGNAL PRESERVE THOSE OF THE RF SIGNAL BUT CONTAIN A PHASE-SHIFT DUE TO THE PHASE OF THE LOCAL OSCILLATOR.



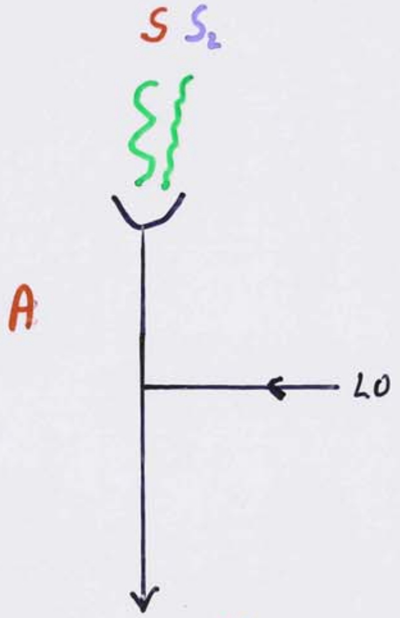

$$\underbrace{V_I}_{} = \underbrace{V_A}_{\text{receiver}} + \underbrace{V_s}_{\text{source}}$$

$$\langle V_I^2 \rangle = \langle (V_A + V_s)^2 \rangle \qquad t > b^{-1}$$

$$= \langle V_A^2 \rangle + \langle V_s^2 \rangle + \langle 2 V_A V_s \rangle$$

$$= \langle V_A^2 \rangle + \langle V_s^2 \rangle$$

$$\propto T_{REC} + T_{ANT}$$




$$V_I = \underset{\text{receiver}}{V_A} + \underset{\text{source}}{V_s}$$

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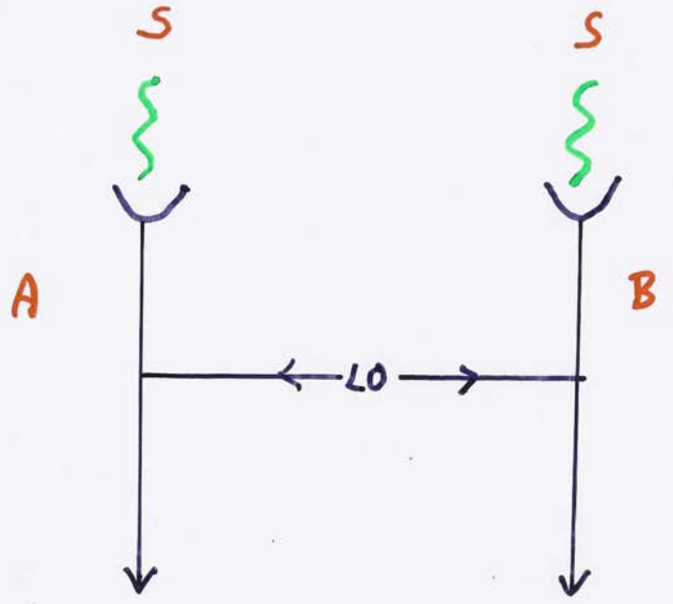
$$= \langle V_A^2 \rangle + \langle V_s^2 \rangle$$

$$\propto T_{REC} + T_{ANT}$$

$$\langle V_I^2 \rangle = \langle (V_A + V_s + V_{S_2})^2 \rangle$$

$$= \langle V_A^2 \rangle + \underbrace{\langle V_s^2 \rangle + \langle V_{S_2}^2 \rangle}$$

$$\propto T_{REC} + T_{ANT}$$




$$V_1 = \underset{\text{receiver}}{V_A} + \underset{\text{source}}{V_S} \qquad V_2 = \underset{\text{receiver}}{V_B} + \underset{\text{source}}{V_S}$$

ADDING INTERFEROMETER (MICHELSON)

$$\langle (V_1 + V_2)^2 \rangle = \langle V_A^2 \rangle + \langle V_B^2 \rangle + 4 \langle V_S^2 \rangle$$

MULTIPLYING INTERFEROMETER

$$\begin{aligned} \langle V_1 . V_2 \rangle &= \langle (V_A V_S + V_A V_B + V_S V_B + V_S^2) \rangle \\ &= \langle V_S^2 \rangle \\ &\propto S \end{aligned}$$

## WITH A PHASE ERROR $\phi_i$

$$V_1 = V_A + V_S \qquad V_2 = V_B + V_S$$

$$V_S = V_S \cos(2\pi f_I) \qquad V_S = V_S \cos(2\pi f_I + \phi_i)$$

$$\langle V_1 . V_2 \rangle = \langle V_S^2 \rangle \cos \phi_i \quad \propto S \cos \phi_i$$

MULTIPLIED OUTPUT DECREASED BY $\cos \phi_i$

## RECOVER FULL POWER WITH SINE AND COSINE MULTIPLIERS

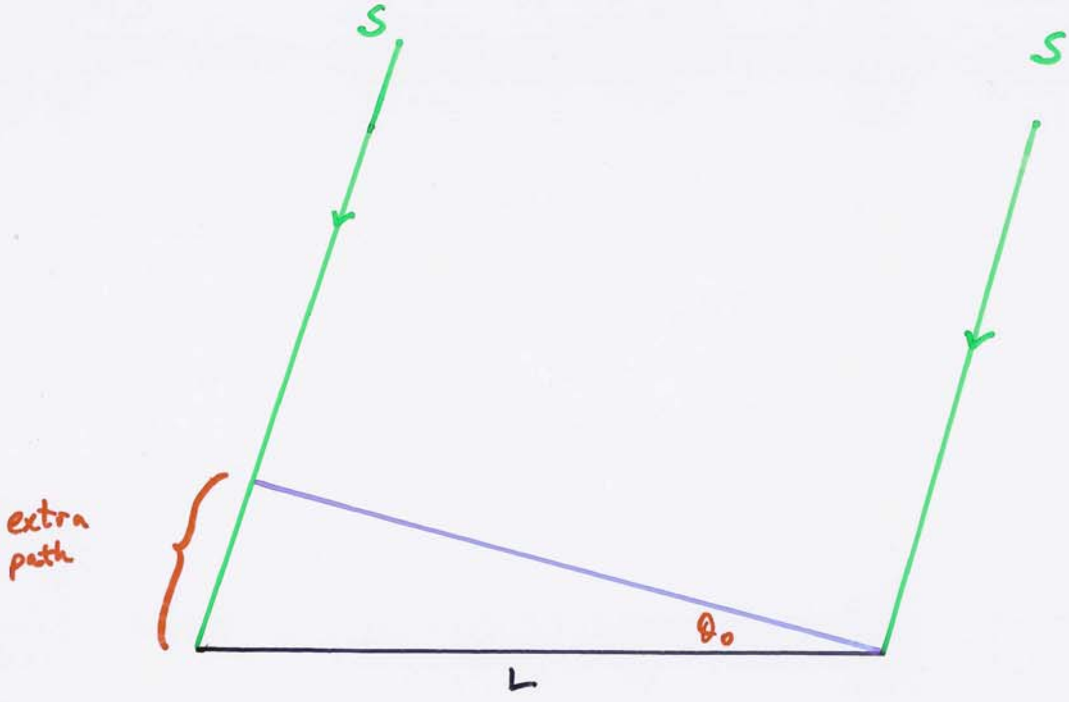
$$V_S = V_S \cos(2\pi f_I) \qquad V_S = V_S \cos\left(2\pi f_I + \phi_i - \frac{\pi}{2}\right)$$

$$\langle V_1 . V_2 \rangle = \langle V_S^2 \rangle \sin \phi_i$$

COSINE AND SINE MULTIPLIERS PRODUCE COMPLEX OUTPUT

$$\langle V_S^2 \rangle e^{i\phi_i} \quad \propto \quad S e^{i\phi_i}$$

IGNORE SUCH PHASE ERRORS FOR NOW ....




EXTRA PATH $= L \sin\theta_0$

EXTRA DELAY, $\tau_c = \dfrac{L\sin\theta_0}{c}$

ADDITIONAL PHASE, $\phi = 2\pi L \dfrac{\sin\theta_0}{c} \cdot f_R = 2\pi \dfrac{L\sin\theta_0}{\lambda_R}$

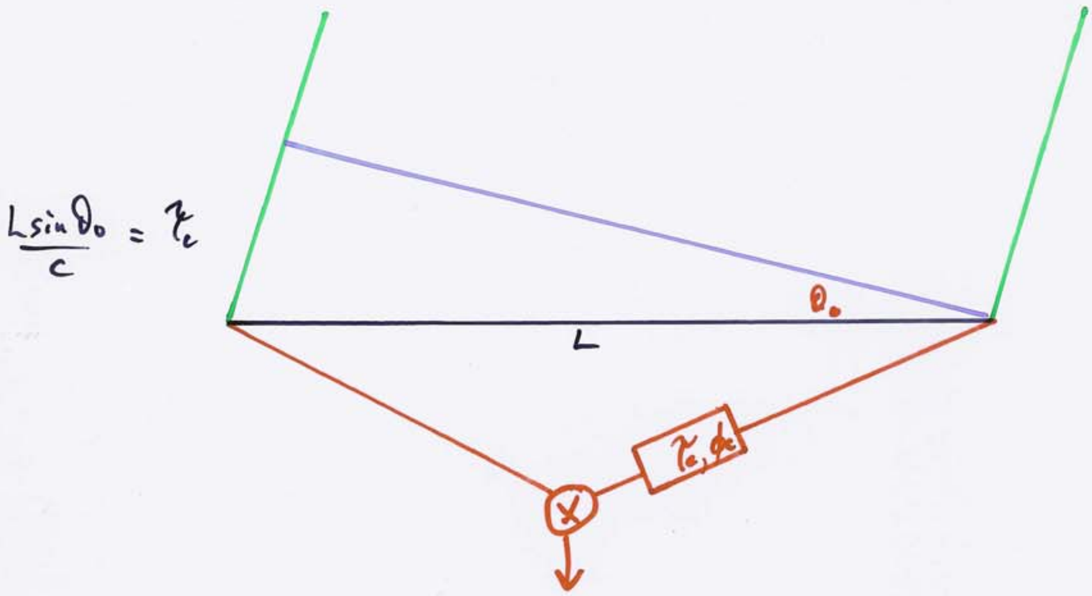
## MULTIPLYING INTERFEROMETER OUTPUT

$\cos x \qquad S_c = S\cos\left(\dfrac{2\pi L\sin\theta_0}{\lambda_R}\right)$

$\sin x \qquad S_s = S\sin\left(2\pi\dfrac{L\sin\theta_0}{\lambda_R}\right)$

$$S = S e^{i\left[2\pi\frac{L\sin\theta_0}{\lambda_R}\right]}$$

RECOVER FULL SIGNAL WITH PATH COMPENSATION




IF PATH DELAY COMPENSATION $\tau_c$

$$\phi_I = 2\pi \frac{L \sin\theta_0}{c} \cdot f_I$$

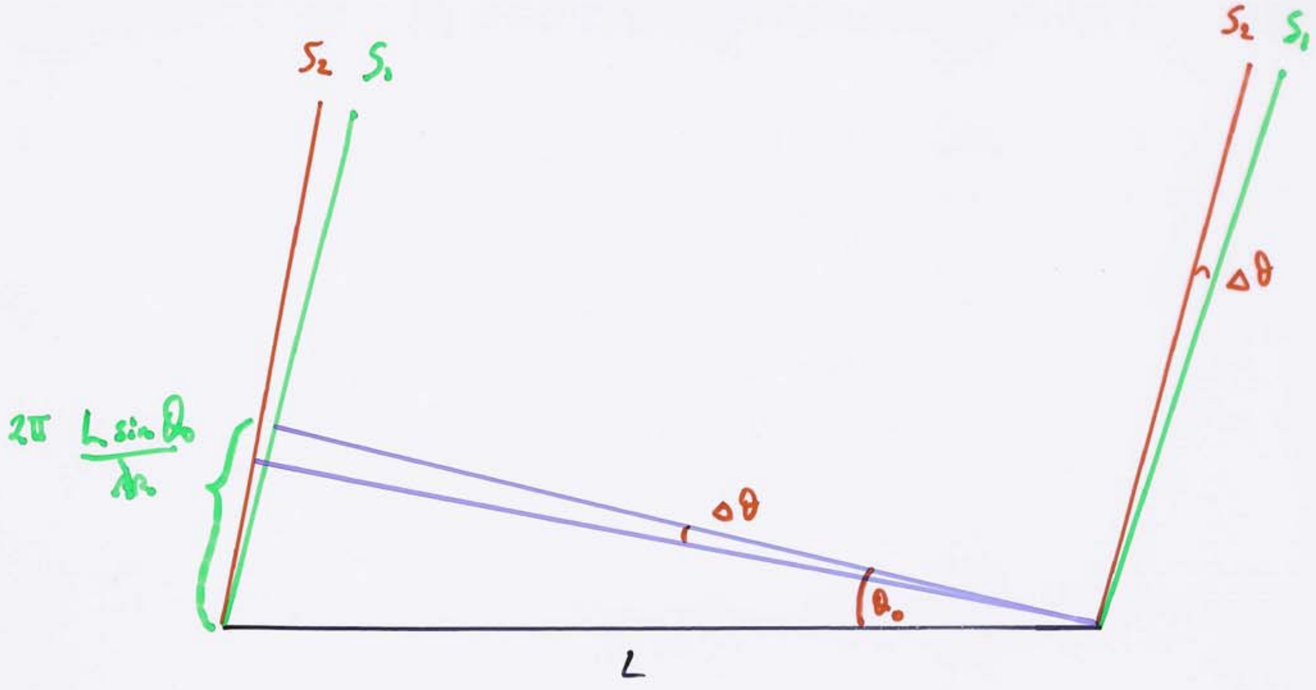
R.F. PHASE PATH

$$\phi_R = 2\pi \frac{L \sin\theta_0}{c} \cdot f_R$$

EXTRA PHASE ROTATION $\phi_c$

$$\phi_c = 2\pi \frac{L \sin\theta_0}{c} \left[ f_R - f_I \right]$$

$$= 2\pi \frac{L \sin\theta_0}{c} \cdot f_c$$




PATH COMPENSATION FOR $S_1$

$$S_c = S_1$$

UNCOMPENSATED PATH FOR $S_2$

$$\Delta\phi = \frac{d}{d\theta}\left[2\pi \frac{L\sin\theta_0}{\lambda_R}\right]\Delta\theta$$

$$\Delta\phi = 2\pi \frac{L\cos\theta_0}{\lambda_R} \cdot \Delta\theta$$

$$\underline{\Delta\phi = 2\pi q \cdot \Delta\theta}$$

$$\boxed{q = \frac{L\cos\theta_0}{\lambda_R}}$$

(units: wavelengths)

$$\underline{S = S_2\, e^{i[2\pi q \Delta\theta]}}$$

$q$ IS A MEASURE OF THE INTERFEROMETER RESOLUTION

- IT IS THE FACTOR BY WHICH ANGULAR CHANGES ON THE SKY ARE MAGNIFIED TO PRODUCE PHASE CHANGES IN THE INTERFEROMETER OUTPUT
- IT IS THE PROJECTED BASELINE LENGTH IN THE DIRECTION OF THE SOURCE, MEASURED IN WAVELENGTHS.

## TOTAL INTERFEROMETER RESPONSE TO $S_1$ AND $S_2$

$$S_{TOT} = S_1 + S_2\, e^{i2\pi q \Delta\theta}$$

## FOR MANY POINT SOURCES

$$S_{TOT} = S_1 + S_2(\Delta\theta_2)\, e^{i2\pi q \Delta\theta_2} \dots + S_i(\Delta\theta_i)\, e^{i2\pi q \Delta\theta_i}$$

## FOR CONTINUOUS DISTRIBUTION

$$dS = b(\Delta\theta)\, d\theta$$

$$S_{TOT} = \int b(\Delta\theta)\, e^{i2\pi q \Delta\theta} \cdot d\theta$$

$$\Theta = \theta - \theta_0$$

$$\boxed{S_{TOT} = \int b(\Theta)\, e^{i2\pi q \Theta}\, d\Theta}$$

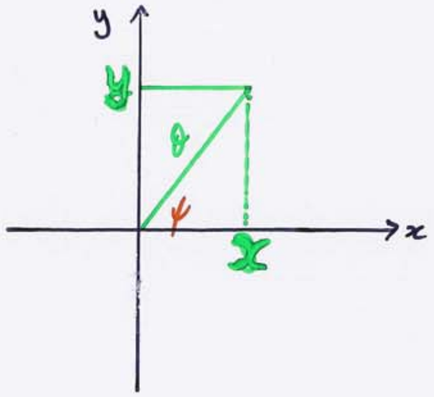
INTERFEROMETER FLUX IS ONE COMPONENT OF THE FOURIER TRANSFORM OF $b(\Theta)$

SPATIAL FREQUENCY $q$

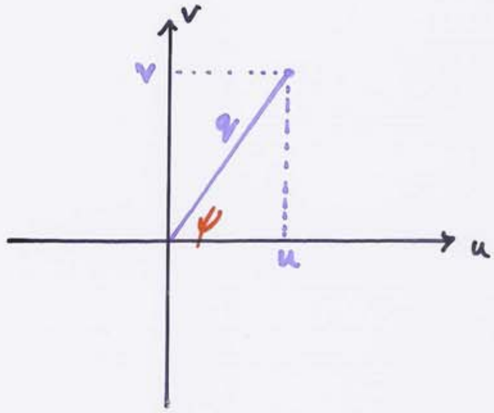
## 2-D FORMALISM

- TELESCOPES A, B AND SOURCE DIRECTION DEFINE AN ARC ON THE SKY

($\vartheta$ and $q$ refer to this direction)




ANGLES ON THE SKY




RESOLUTION ON THE SKY

$$x = \vartheta \cos\psi$$
$$y = \vartheta \sin\psi$$

$$u = q \cos\psi$$
$$v = q \sin\psi$$

$$S_{TOT} = \int b(x,y)\, e^{i2\pi(ux+vy)}\, dx\, dy$$

# EFFECT OF EARTH ROTATION

$$\theta_0 = \theta_0(t)$$

RESOLUTION CHANGES WITH TIME

$$\frac{L\cos\theta_0}{\lambda_R} = q = q(t)$$

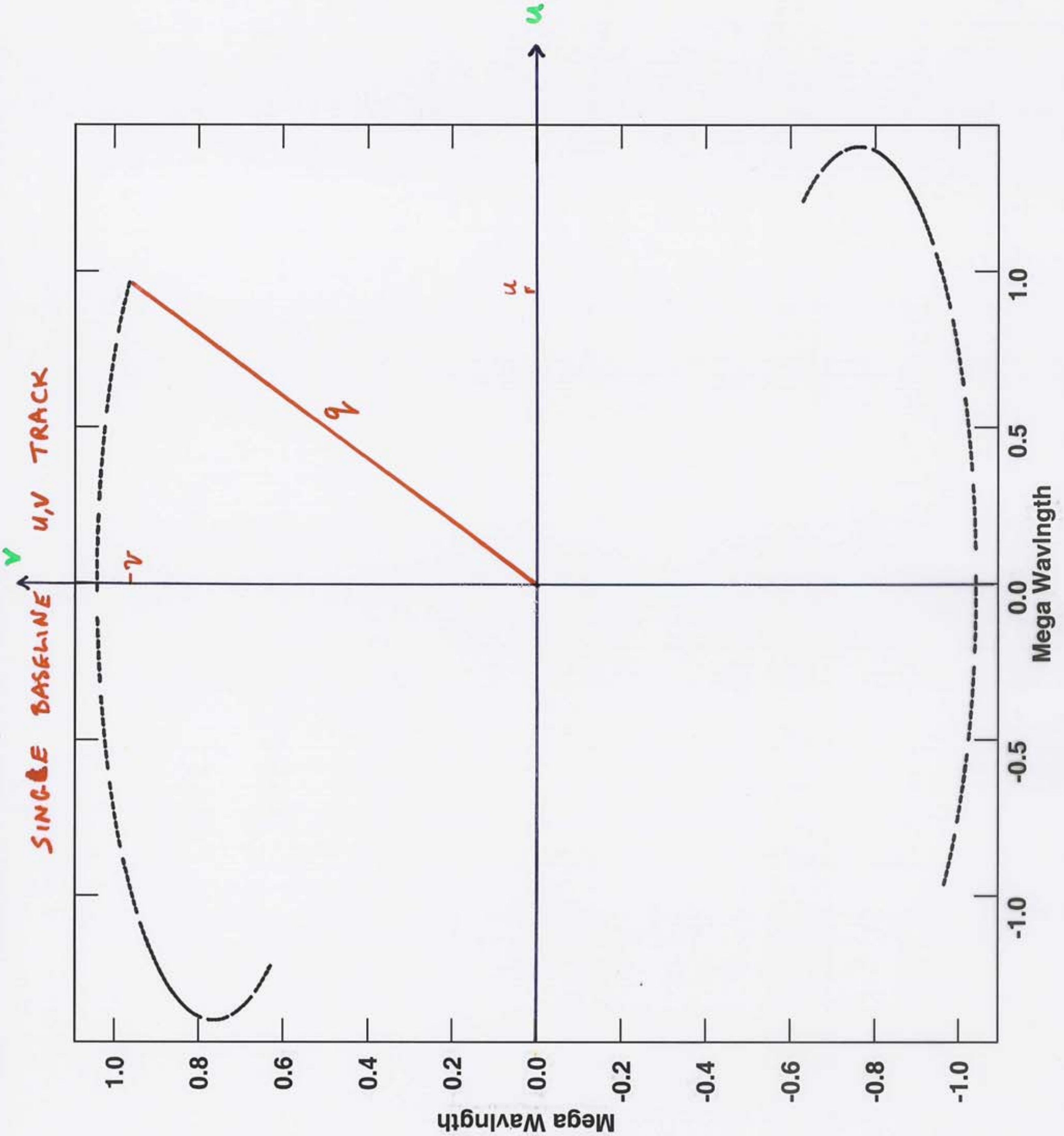
$u(t)$
$v(t)$

$u, v$ track with time traces an ellipse

REQUIRED PATH COMPENSATION CHANGES WITH TIME

$$\frac{\partial}{\partial t}\left[\frac{L\sin\theta_0}{c}\right] = \frac{\partial \tau_c}{\partial t}$$ DELAY TRACKING (AT IF)

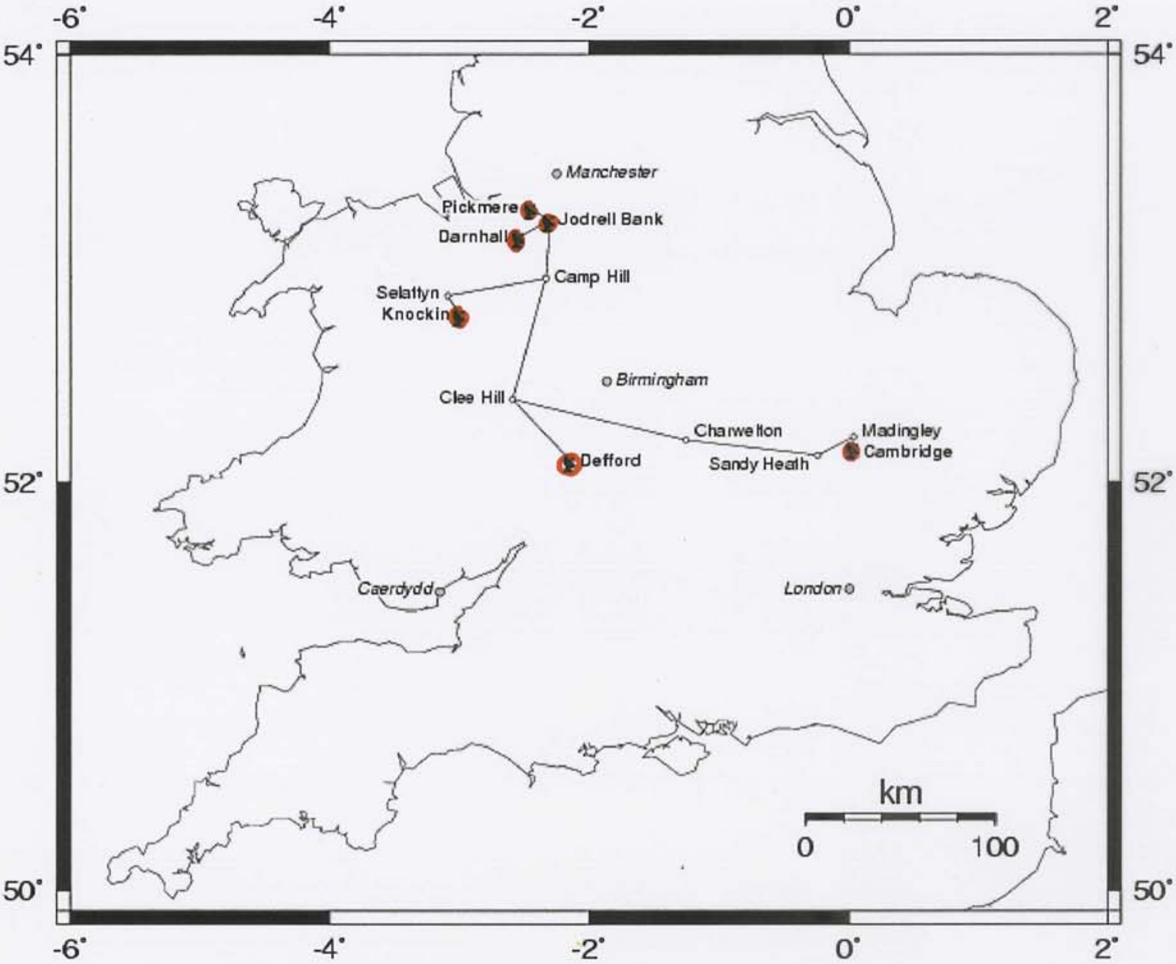
$$\frac{\partial}{\partial t}\left[2\pi\frac{L\sin\theta_0}{c}\cdot f_L\right] = \frac{\partial \phi_c}{\partial t}$$ FRINGE ROTATION


SINGLE BASELINE u,v TRACK
v
u
-v
u
q
Mega Wavlngth
Mega Wavlngth
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.0
-0.5
0.0
0.5
1.0

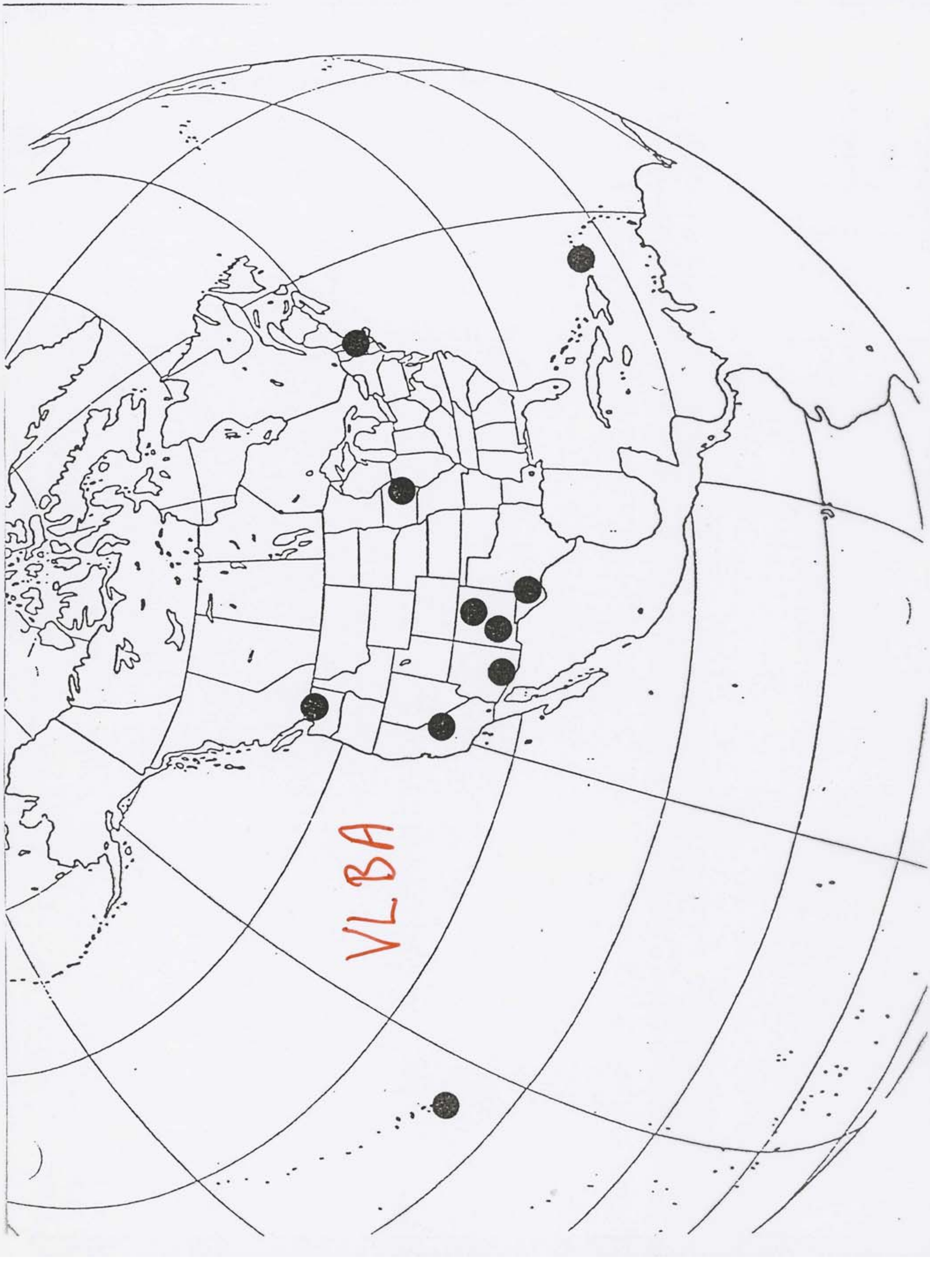
MERLIN

# MERLIN

Merlin stands for the Multi-Element Radio-Linked Interferometer Network,Jodrell Banks array of six observing stations that together form a powerful telescope with an effective aperture of over 217 kilometres.




At a wavelength of six centimeters MERLIN has a maximum resolution of 40 milli arc seconds, about twenty times better than can commonly be achieved by the best ground-based telescopes, and comparable to the Hubble Space Telescope. Such resolving power is equivalent to measuring the diameter of a one-pound coin from a distance of 100 kilometres.


VLBA

# MULTI- ANTENNA INTERFEROMETER ARRAYS

| ARRAY | TELESCOPES | BASELINES |
|---|---|---|
| MERLIN | 6 | 15 |
| VLA | 27 | 351 |
| VLBA | 10 | 45 |

MERLIN

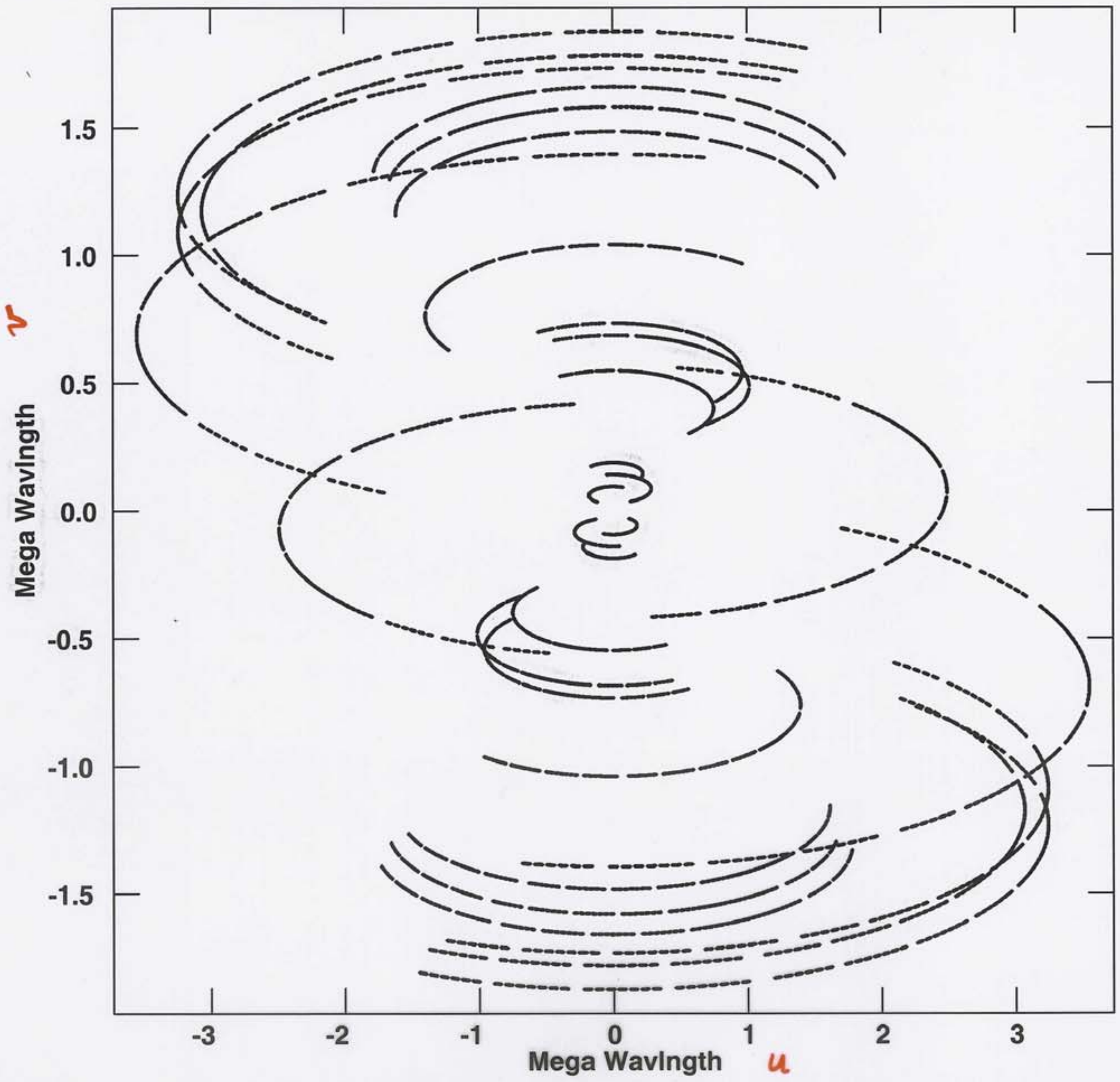
15 BASELINES

15 SIMULTANEOUS MEASUREMENTS OF DIFFERENT FOURIER COMPONENT $q_i$
$(u_i\ v_i)$

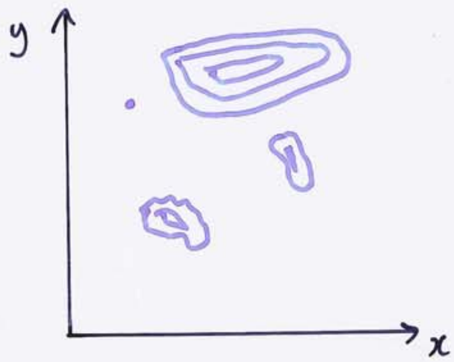
15 TRACKS IN uv PLANE (uv COVERAGE)
(SAMPLING FUNCTION)

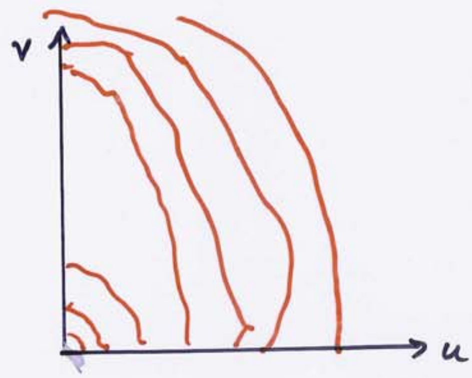
AFTER A (e.g.) 12-h SOURCE TRACK

U,V COVERAGE SAMPLING FUNCTION



## FROM FOURIER COMPONENTS TO SKY MAP




$b(x,y)$ FT $S(u,v)$

## WITH FULL U,V COVERAGE

$S(u,v)$ FT $b(x,y)$

## WITH INCOMPLETE U,V COVERAGE

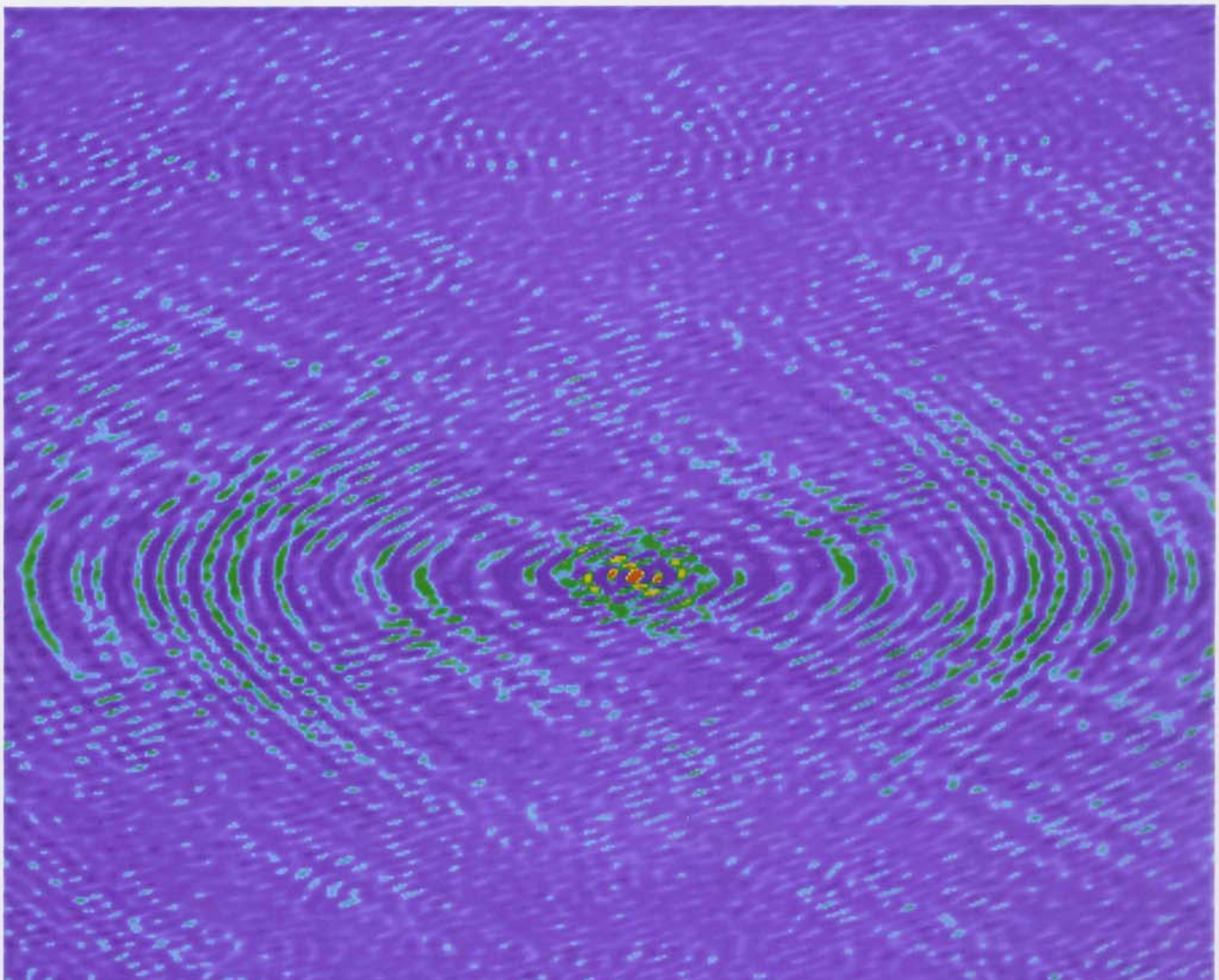
SAMPLE FUNCTION $T(u,v)$ (= u,v coverage)

$$S(u,v) \cdot T(u,v) \quad \text{FT} \quad \tilde{T}(x,y) \otimes b(x,y)$$

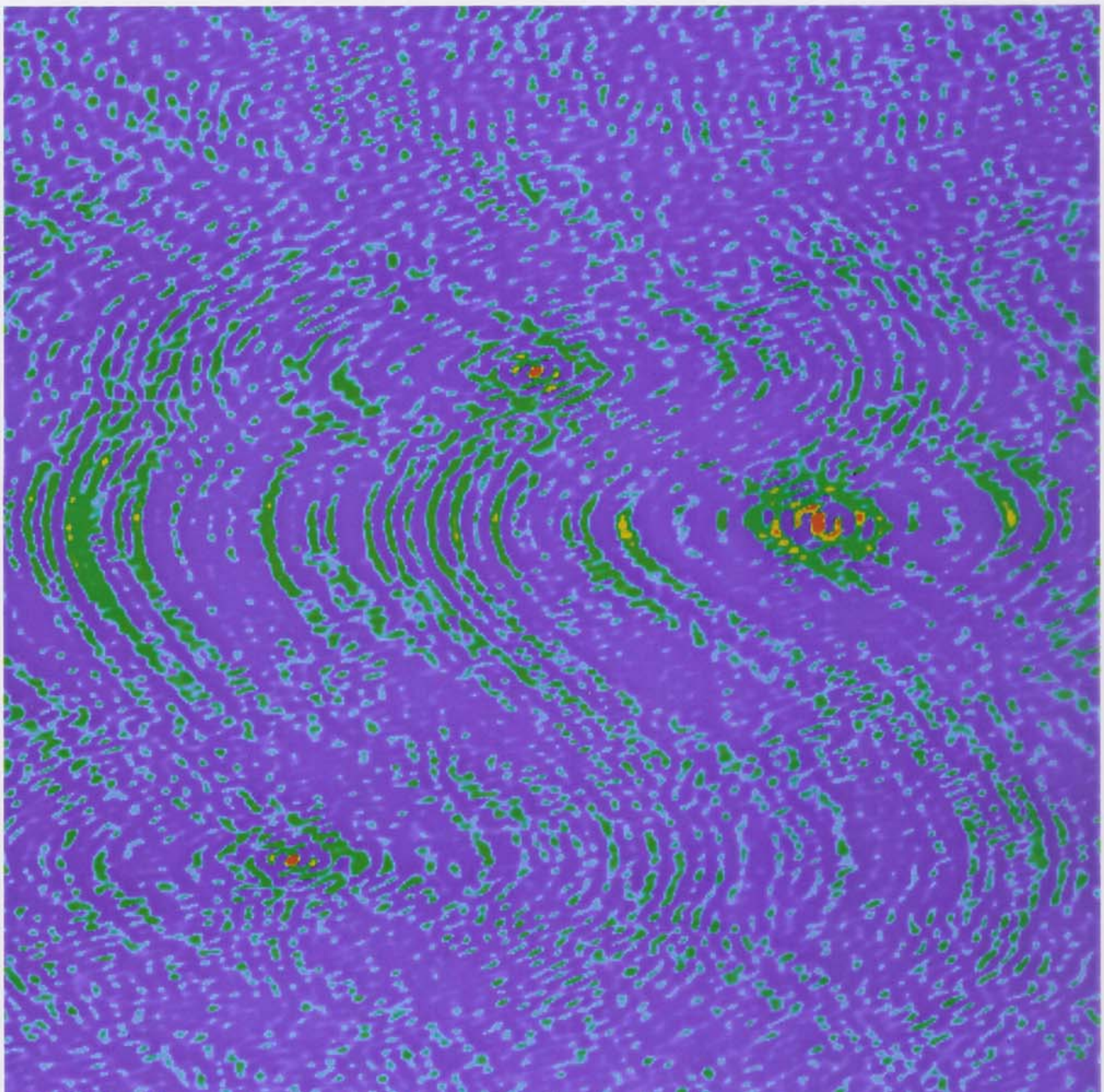
data FT sky convolved with $\tilde{T}$ = "DIRTY MAP"

$\tilde{T}(x,y)$ is KNOWN AS THE "DIRTY BEAM"

= RESPONSE TO A POINT SOURCE



DIRTY BEAM



DIRTY MAP

# DECONVOLUTION ALGORITHM "CLEAN"

- COMPUTE DIRTY BEAM $T(u,v) \;\; \text{FT} \;\; \tilde{\Upsilon}(x,y)$
- COMPUTE DIRTY MAP $S(u,v) \cdot T(u,v) \;\; \text{FT} \;\; \tilde{\Upsilon}(x,y) * b(x,y)$

(•) TRY TO RECOGNIZE DIRTY BEAM RESPONSE IN DIRTY MAP

- LOCATE POSITION OF BRIGHTEST POINT IN DIRTY MAP
- SUBTRACT SCALED VERSION OF DIRTY BEAM CENTRED AT THAT POINT
  - SCALED BY DIRTY MAP VALUE
  - SCALED BY $L \times$ (DIRTY MAP VALUE)
    $L$ = "LOOP GAIN" $\sim 0.1$
- STORE THE VALUE AND LOCATION OF THE POINT-SOURCE RESPONSE REMOVED
  = CLEAN COMPONENT $CC(x,y)$

- REPEAT LAST 3 STEPS UNTIL THE BRIGHTEST POINT IN THE DIRTY MAP IS BELOW THE MAP NOISE LEVEL

- WE ARE LEFT WITH A DENUDED DIRTY MAP (= "RESIDUAL MAP") AND A LIST OF CLEAN COMPONENTS.
  RESIDUAL MAP IS HOPEFULLY JUST NOISE.

- THE $CC(x,y)$ REPRESENT A MULTI-POINT-SOURCE MODEL OF $b(x,y)$ WHICH, WHEN CONVOLVED WITH THE DIRTY BEAM, WOULD REPRODUCE THE DIRTY MAP.

- IN GENERAL A POOR REPRESENTATION OF A CONTINUOUS BRIGHTNESS DISTRIBUTION $b(x,y)$

- INSTEAD WE CONVOLVE THE $CC(xy)$ WITH AN IDEALIZED BEAM FUNCTION – THE CLEAN BEAM

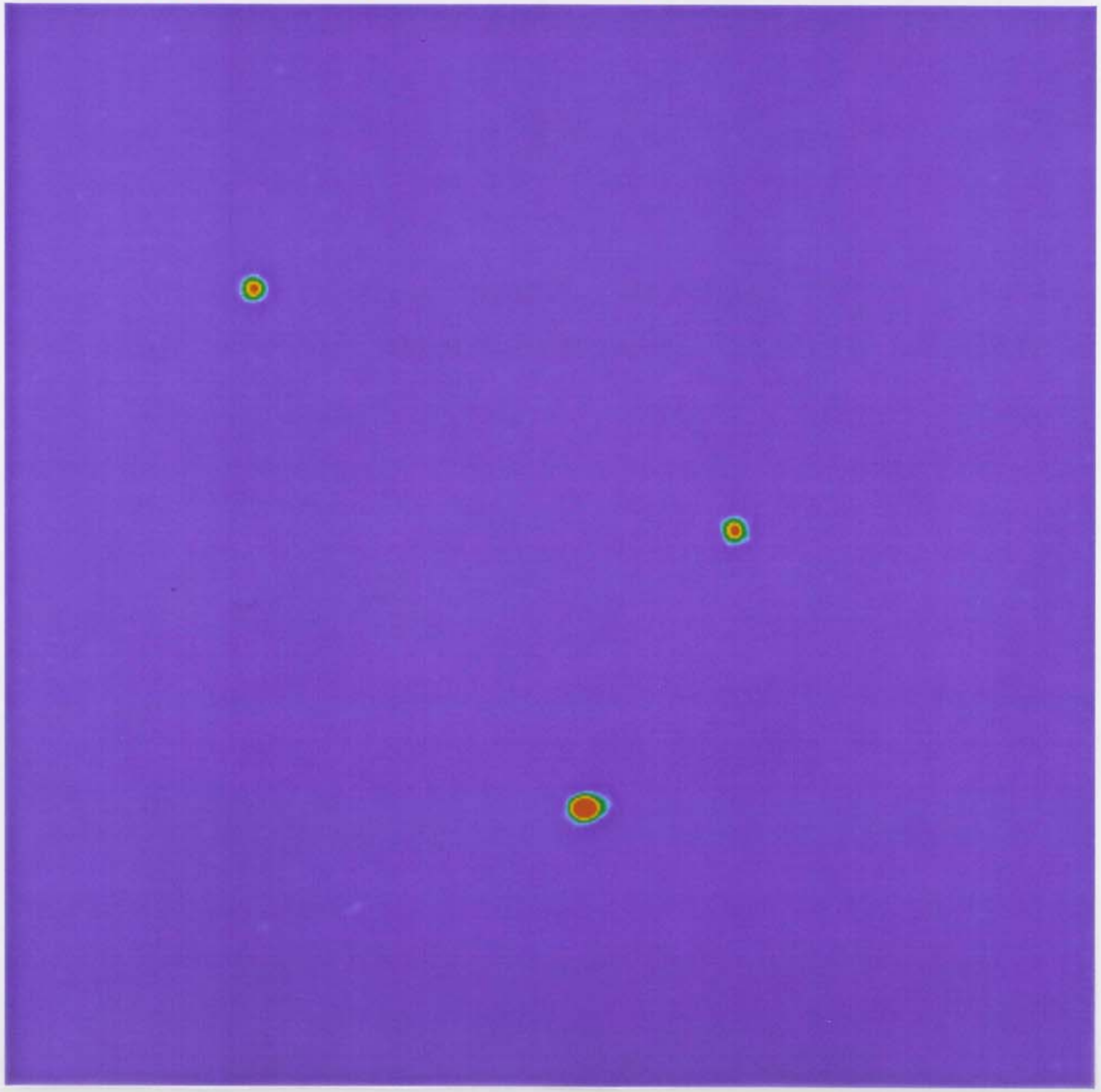
- CLEAN BEAM SIZE CHOSEN TO CORRESPOND TO THE DIRTY BEAM RESOLUTION (CENTRAL PEAK REGION OF DIRTY BEAM)

- ADD TO RESIDUAL MAP TO INDICATE NOISE

$$\underbrace{CM(x,y)}_{\text{clean map}} = \underbrace{CB(x,y)}_{\text{clean beam}} \circledast \underbrace{CC(x,y)}_{\text{clean components}} + \underbrace{RM(x,y)}_{\text{residual map}}$$

CLEAN MAP CLEAN BEAM 0".071 × 0".044 PA 31°

CLEAN MAP CLEAN BEAM 0·1" x 0·1"



## INSTRUMENTAL PHASE ERRORS

$$S_{ij} = \int b(\theta) . e^{i2\pi q_{ij}\theta} . d\theta$$

$$S_{ij} = a_{ij} , \phi_{ij}$$

$$S_{ij} = a_{ij} , \phi'_{ij} \qquad \phi'_{ij} = \phi_{ij} + \phi^{e}_{ij}$$

## CALIBRATION ON POINT SOURCE OF KNOWN POSITION

$$S^{CAL}_{ij} = a^{CAL}_{ij} , \underset{= 0}{\phi^{CAL}_{ij}}$$

INTERFEROMETER PHASE MEASURED ON CALIBRATOR IS THE ERROR PHASE $\phi^{e}_{ij}$

# PHASE SELF-CALIBRATION

A bit of a fiddle ?....

.... or a powerful algorithm ?

- BASED ON A SIMPLE IDEA
- WE OBSERVE A SOURCE WITH N ANTENNAS $N \geq 3$
- AT TIME $t$ WE HAVE $\frac{N(N-1)}{2}$ BASELINES OF DATA
- THE MEASURED VISIBILITY PHASES, $\phi'_{ij}$ ARE CORRUPTED VERSIONS OF THE TRUE PHASES, $\phi_{ij}$
- ASSUME THAT THE CORRUPTING PHASES ARISE AT TELESCOPES, $\phi_i$, AND CONTRIBUTE TO ALL BASELINES TO ANY TELESCOPE IN THE SAME WAY
- $\underbrace{\phi'_{ij}}_{\text{measured}} = \underbrace{\phi_{ij}}_{\text{true}} + \underbrace{\phi_i - \phi_j}_{\text{antenna corruptions}}$

- NOW THE TRICK ......
- SUPPOSE WE KNOW THE TRUE VISIBILITY PHASES, $\phi_{ij}$ ( because we know the source structure and can thus compute them for each baseline for time t )
- THEN WE CAN SOLVE THE $\frac{N(N-1)}{2}$ SIMULTANEOUS EQUATIONS TO DETERMINE THE CORRUPTING PHASES, $\phi_i$, BECAUSE THERE ARE MORE EQUATIONS THAN UNKNOWNS

- $$\left.\begin{aligned}
\phi'_{12} &= \phi_{12} + \phi_1 - \phi_2 \\
\phi'_{13} &= \phi_{13} + \phi_1 - \phi_3 \\
\phi'_{14} &= \phi_{14} + \phi_1 - \phi_4 \\
\phi'_{23} &= \phi_{23} + \phi_2 - \phi_3 \\
&\vdots
\end{aligned}\right\} \frac{N(N-1)}{2} \text{ equations}$$

measured calculated unknowns (N)

- $\frac{N(N-1)}{2} \geq N$ SO SYSTEM IS OVER-DETERMINED : USE LEAST SQUARES MINIMIZATION

- BUT ...... WE DO NOT KNOW THE SOURCE STRUCTURE YET ..... SO ....

# THE ALGORITHM

- MAKE A GUESS FOR THE SOURCE STRUCTURE
  (Point source, .... )

- CALCULATE SOURCE VISIBILITY PHASE FOR EACH BASELINE (FOR EACH TIME $t$)

CALIB

- SOLVE THE EQUATIONS FOR THE $\phi_i$ FOR EACH TIME $t$

- CORRECT THE MEASURED VISIBILITY FUNCTION

$$\phi_{ij}^{c} = \phi_{ij}' - \phi_i + \phi_j$$

- MAKE A MAP OF THE SOURCE USING THE CORRECTED VISIBILITIES
  IMAGR CLEAN

- USE THIS NEW SOURCE STRUCTURE AS A BETTER INPUT GUESS

- HOPE THAT THIS CONVERGES TO

$$\underset{\text{corrected}}{\phi_{ij}^{c}} = \underset{\text{true}}{\phi_{ij}}$$

- THE ALGORITHM IS KNOW AS "HYBRID MAPPING"

- IT IS BASED ON PHASE SELF-CALIBRATION

- IT ONLY WORKS IF THERE IS GOOD SNR FOR THE VISIBILITIES

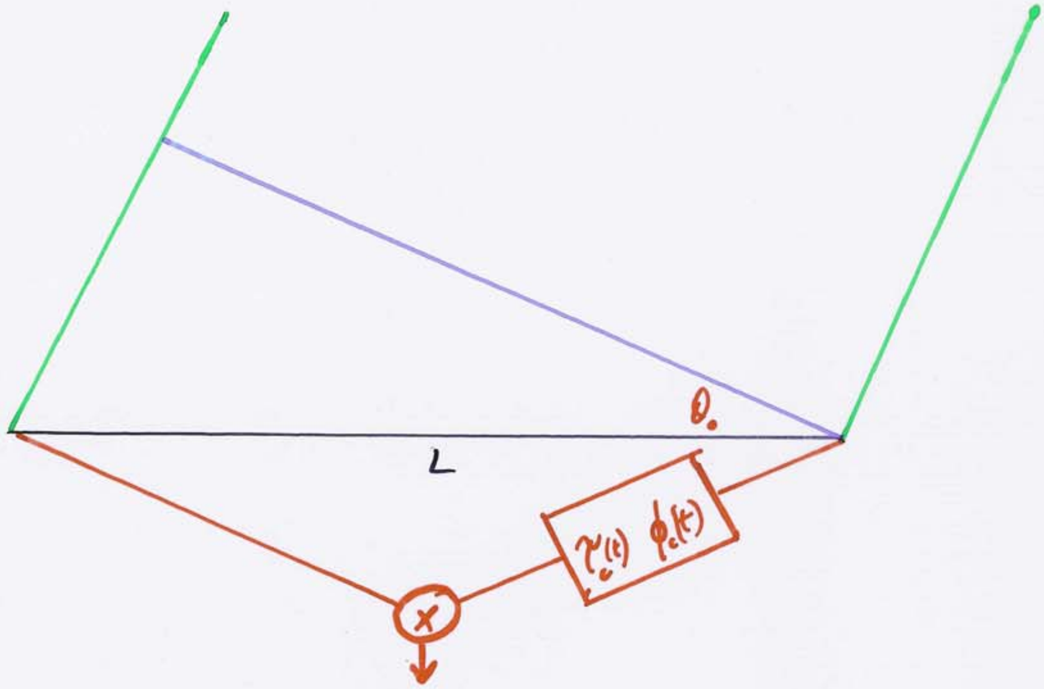
- IT DESTROYS THAT PART OF THE PHASE INFORMATION WHICH GIVES THE POSITION OF THE SOURCE

- THE POSITION OF A SOURCE IN A HYBRID MAP SIMPLY REFLECTS THE POSITION OF THE INITIAL MODEL GUESS (e.g. POINT AT THE MAP ORIGIN)

- ONLY N-1 VALUES OF $\phi_i$ CAN BE DETERMINED; WE SET THAT OF A REFERENCE ANTENNA TO ZERO ($\phi_{ref} = 0$) SINCE FOR ANY SOLUTION SET $\phi_i$, THE VALUES $\phi_i + k$ ARE ALSO A SOLUTION SET

# IMPERFECT PATH COMPENSATION

$\tau = \frac{L \sin\theta_0}{c}$




IF DELAY COMPENSATION $\tau_c(t) = \frac{L \sin\theta_0(t)}{c}$

IF PHASE ROTATION $\phi_c(t) = 2\pi \frac{L \sin\theta_0(t)}{c} \cdot f_L$

$\tau_c$, $\phi_t$ CALCULATION DEPENDS ON KNOWLEDGE OF $L$, $\theta_0$

source position $(\alpha, \delta)$
sidereal time
baseline vector (antenna coordinates)
[VLBI: clock errors]

- FOR LONG BASELINES THE EFFECT OF ANGULAR ERRORS IN THE MODEL PRODUCE LARGER ERRORS IN $\tau_c(t)$, $\phi_c(t)$.

EXAMPLE

| $\delta\theta$ | | | $\delta L \sin\theta$ | |
|---|---|---|---|---|
| $1''$ | $L = 5\,km$ | $\lambda = 6\,cm$ | $\rightarrow$ 2.4 cm | $(144°)$ |
| $1''$ | $L = 5000\,km$ | $\lambda = 6\,cm$ | $\rightarrow$ 24 m<br>$(\delta\tau_c = 80\,ns)$ = residual delay | |

c.f. coherence time for $b = 20\,MHz$ = $50\,ns$

$\Rightarrow$ LOSS OF COHERENCE

$\Rightarrow$ NO OUTPUT FROM MULTIPLIER

- OVERCOME WITH "MULTI-LAG" INTERFEROMETER

$$\langle V_1 . V_2 \rangle$$

$$\langle V_1(t) . V_2\left(t + \tfrac{1}{2b}\right) \rangle$$

$$\langle V_1(t) . V_2\left(t + \tfrac{2}{2b}\right) \rangle$$

$$\vdots$$

$$\langle V_1(t) . V_2\left(t + \tfrac{i}{2b}\right) \rangle \qquad i = -16 \rightarrow 16$$

- NEED TO CHOOSE WHICH VALUE OF $i$ HAS THE SIGNAL

- SIGNAL MUST BE STRONG ENOUGH TO RECOGNIZE (e.g. S/N-ratio > 5)

- WE MAY NEED TO INTEGRATE THE SIGNAL TO ACHEIVE SUFFICIENT S/N-ratio.

- BUT ERRORS IN MODEL PRODUCE ERRORS IN FRINGE ROTATION PHASE, $\phi_c$, WHICH CHANGE WITH TIME AND PREVENT (VECTOR) INTEGRATION

EXAMPLE

| $\delta\theta$ | | | $\delta\dot{\phi}_c$ | |
|---|---|---|---|---|
| $1''$ | $L = 5000$ km | $\lambda = 6$ cm | 33 mHz | (turn is $30^s$) |

= residual fringe rate

- NEED TO TRY DIFFERENT RESIDUAL FRINGE RATES TO FIND WHICH IS THE CORRECT ONE!

## FRINGE - FITTING (FRING)

- SEARCH TO FIND THE SIGNAL IN INTEGRATION TIME $t$ (SOLINT)
- SIMULTANEOUS SEARCH OF RESIDUAL DELAY SAMPLES $(i = -16 \rightarrow 16)$ AND A RANGE OF POSSIBLE RESIDUAL FRINGE-RATES
- SIGNAL MUST BE ABOVE A DETECTION THRESHOLD
- SEARCH IDENTIFIES CORRECT RESIDUAL DELAY AND FRINGE-RATE