Radiative signatures of Fermi acceleration at relativistic shocks

Brian Reville

Heidelberg Joint Astronomical Colloquium,
January 18, 2011

Outline

Relativistic flows in Astrophysics

Fermi acceleration at non-relativistic shocks

Fermi acceleration at relativistic shocks

Radiative signatures of Fermi acceleration

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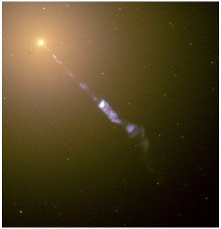
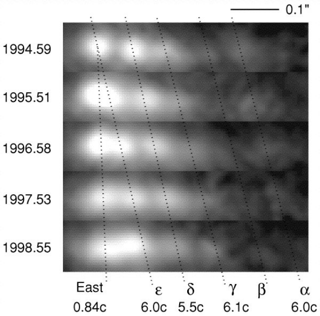
Apparent superluminal motion

Hubble image of M87

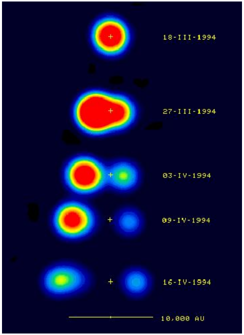




Biretta et al. ’99

$\beta_{app} \approx 6$

Microquasar GRS1915+105



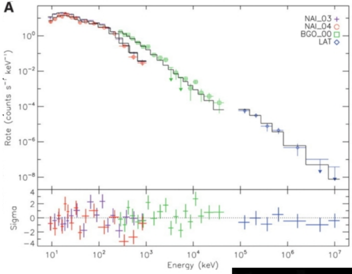
Mirabel & Rodriguez ’94

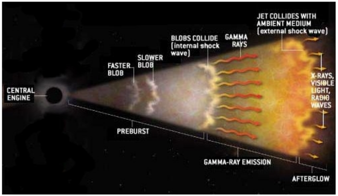
$\beta_{app} = 1.25 \pm 0.15, 0.65 \pm 0.08$

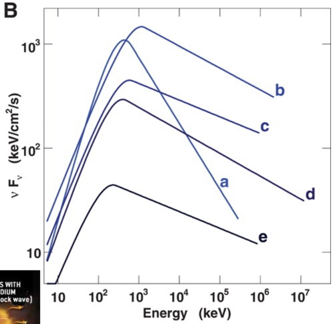
$$\beta_{app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

GRBs – γ ray transparency

GRB 080916C





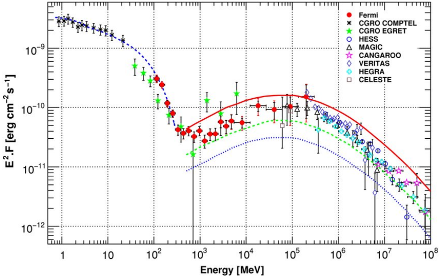


No cut-off/γ-ray attenuation, $\tau_{\gamma\gamma}(\Gamma) < 1$

For GRB 080916C: $\Gamma_{\rm min} \gtrsim 10^3$ (Abdo et al. '09)

Pulsars - termination shock

Credit: J. Hester (ASU), CXC, HST, NRAO, NSF, NASA



Abdo et al. 2010

Ultrarelativistic $\Gamma \gg 1$ wind launched near pulsar light-cylinder
Crab: $\Gamma_T \sim 10^3 - 10^6$ (Kennel & Coroniti '84)

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Fermi acceleration at non-relativistic shocks

Fermi acceleration at relativistic shocks

Radiative signatures of Fermi acceleration

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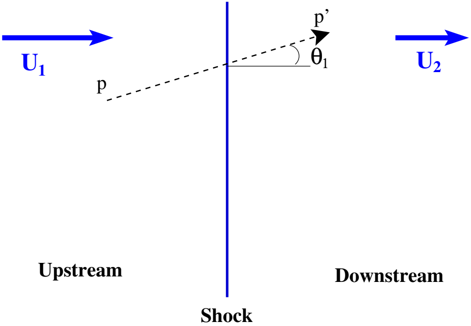
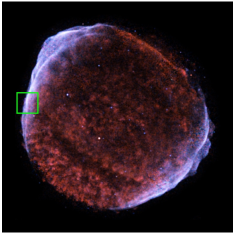
Bell 78, Krymskii 77, etc. $U_1, U_2 \ll v \sim c$





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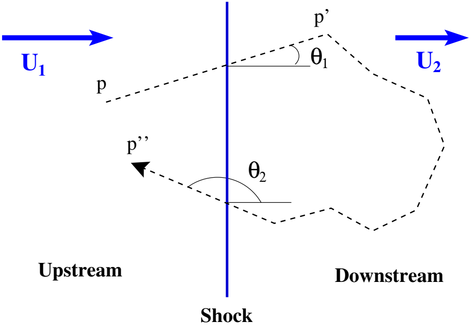

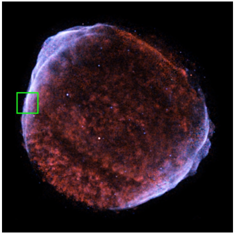

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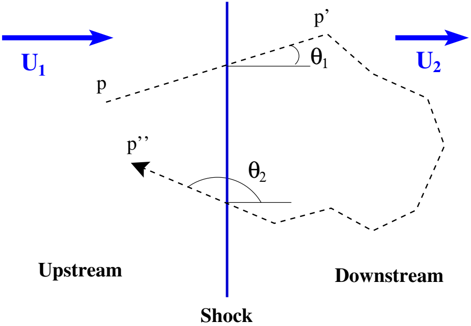

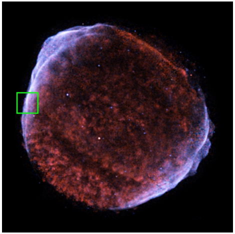

average momentum increase per cycle (to first order in U/v)

$$\frac{\langle \Delta p \rangle}{p} = \frac{4}{3} \frac{U_1 - U_2}{v}$$

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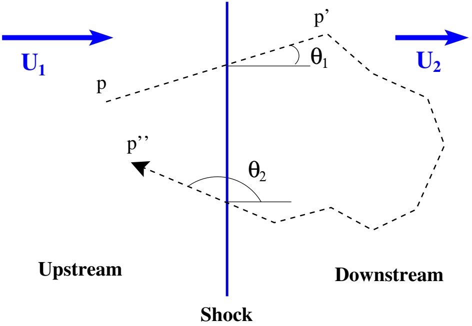

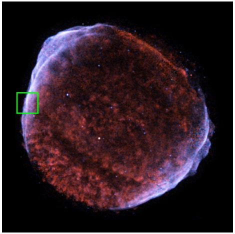

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$$\frac{\langle \Delta p \rangle}{p} = \frac{4}{3} \frac{U_1 - U_2}{v}$$

$$\text{Escape Probability} = \frac{\text{flux to downstream}}{\text{flux across shock}} = \frac{4U_2}{v}$$

Fermi acceleration at non-relativistic shocks

- Fractional increase in momentum versus fractional escape:

$$N_>(p + \langle \Delta p \rangle) = N_>(p)(1 - P_{\rm esc})$$

where $N_>(p) = \int_p^\infty n(p')dp'$

- Naturally produces power-law

$$n(p) \propto p^{-1-P_{\rm esc}/(\langle \Delta p \rangle /p)} \propto p^{-(r+2)/(r-1)}$$

- the shape of the power law depends only on a single parameter

$$r = \frac{U_1}{U_2} = \frac{\gamma + 1}{\gamma - 1 + 2/M_1^2}$$

independent of the scattering mechanism!!

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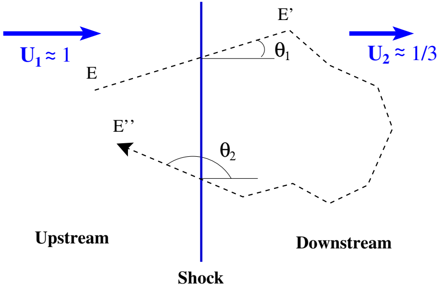
- For strong shocks $r \to 4$, produces spectrum $n(p) \propto p^{-2}$

Outline

Particle kinematics at relativistic shocks



Ultrarelativistic limit:

$\Gamma_{\rm sh} \rightarrow \infty \ (U_1 \rightarrow 1)$

$\Gamma_{\rm rel} = \Gamma_{\rm sh}/\sqrt{2}$

$U_2 = \frac{1}{3}$

i.e. shock velocity mildly rel. in the downstream frame

$$\frac{E''}{E} = \frac{1}{2}\Gamma_{\rm sh}^2 \left[1 + \beta_{\rm rel}\cos(\bar{\theta}_1)\right]\left[1 - \beta_{\rm rel}\cos(\widetilde{\theta_2})\right]$$

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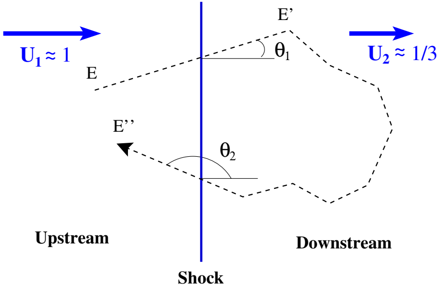

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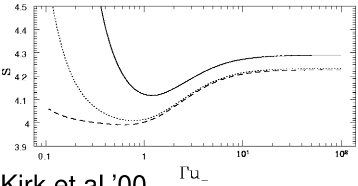
- to overtake shock (in ds frame) $|\cos(\widetilde{\theta}_2)| > U_2 \approx 1/3$
- upstream frame: particles inside narrow cone $\bar{\theta} < 1/\Gamma_{sh}$
- particle overtaken by shock once $\bar{\theta} > 1/\Gamma_{sh}$
- net energy gain: $\frac{E''}{E} \sim 2$

Fermi acceleration - theory & simulations

Semi-analytic theory:

- e.g. Kirk & Schneider '87,
  Heavens & Drury '88,
  Kirk et al '00,
  Dempsey & Kirk '08
- solve the transport eqn.
  $\Gamma(u+\mu)\frac{\partial f}{\partial z} = \frac{\partial}{\partial\mu}\left(D(1-\mu^2)\frac{\partial f}{\partial\mu}\right)$



Kirk et al '00

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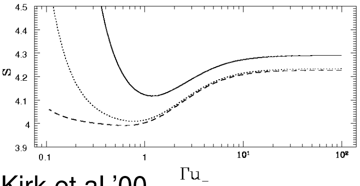
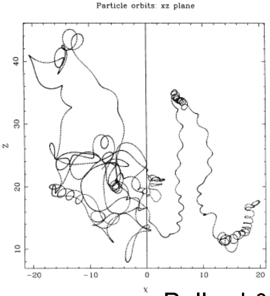


Kirk et al '00

Monte-Carlo simulations:

- e.g. Kirk & Schneider '87, Ballard & Heavens '92, Bednarz & Ostrowski '98 Achterberg et al '01, Lemoine et al. '06



Ballard & Heavens '92

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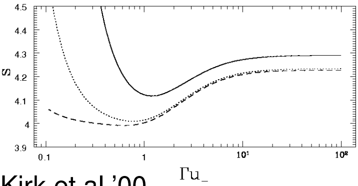

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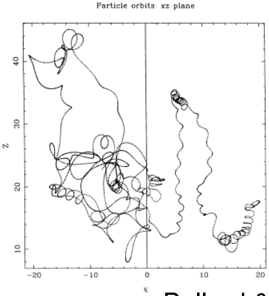
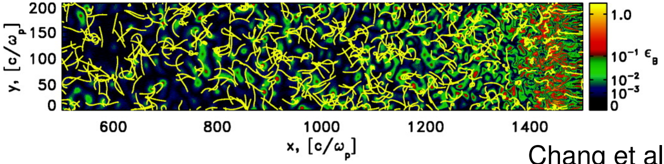


Ballard & Heavens '92

- Power-law index sensitive to details of scattering
- Strong turbulence necessary to provide pitch-angle scattering and cross field diffusion

Particle in Cell simulations of relativistic shocks

- PIC simulations track particle motion and calculate fields *'self-consistently'*



Chang et al. 08

- Does Fermi acceleration operate at these shocks?

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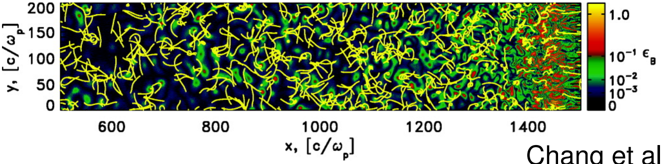

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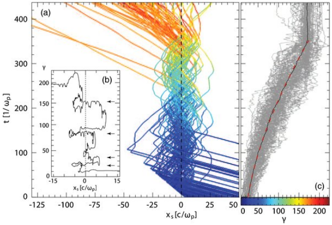
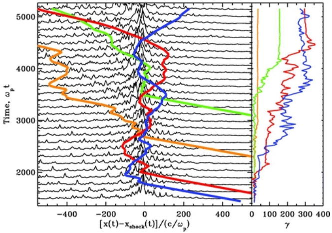
Martins et al. '09



Spitkovsky '08



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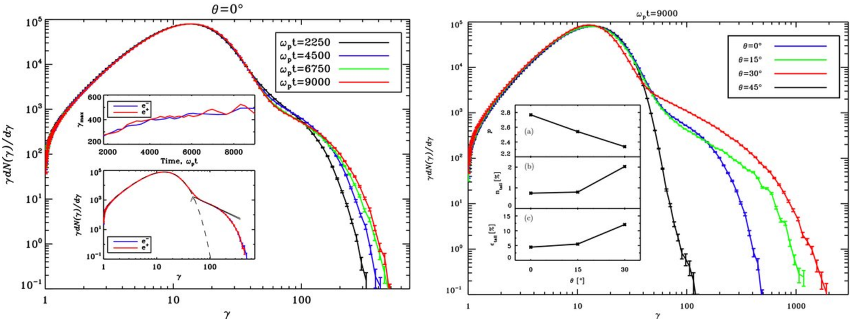


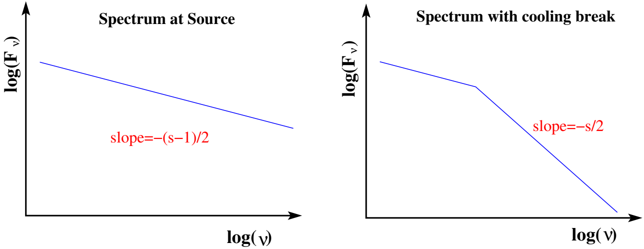
Sironi & Spitkovsky '09

- Fermi acceleration found to occur in unmagnetised and subluminal shocks.
- Superluminal shocks unable to generate the strong turbulence (essential for Fermi acc., Ostrowski et al.)

Outline

Radiative signatures of Fermi acceleration

- $n(p) \propto p^{-s}$ produces power-law synchrotron spectrum



- simulations & theory for ultra-relativistic shock suggest $s \approx 2.2 - 2.7$ (Achterberg et al. '00, Lemoine & Revenu '06, etc.)
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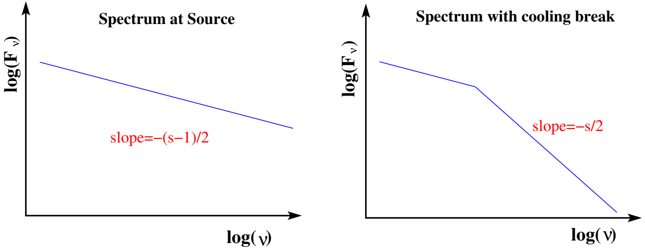

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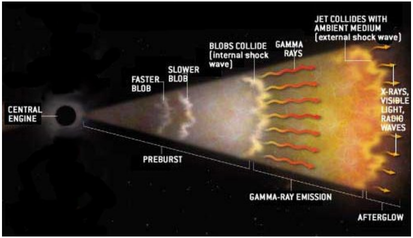
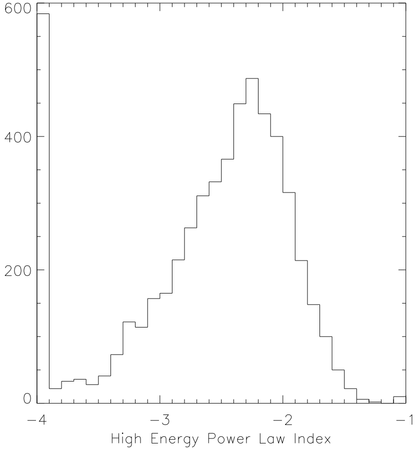
- current PIC simulations suggest $s \sim 2.3 - 2.8$
- hard x-ray spectrum in Crab nebula $F_\nu \propto \nu^{-1.1}$
- produced by synchrotron cooled $s = 2.2$ injection spectrum

Radiative signatures of Fermi acceleration

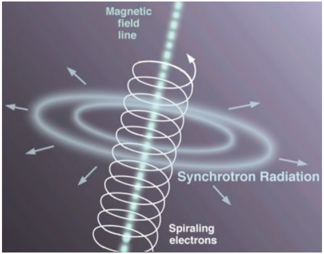
- Histogram of high energy photon index for BATSE detected GRBs ($\epsilon > h\nu_b$, ν_b=peak in νF_ν)
- $n(\epsilon) \propto \epsilon^\beta$ ($F_\nu \propto \nu^{\beta+1}$) Preece et al '00





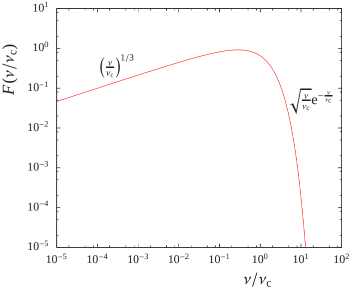
- histogram peaks at $\beta \approx -2.25$ ($F_\nu \propto \nu^{-1.25}$)
- consistent with synchrotron cooled $s = 2.5$ injection spectrum

Synchrotron radiation



$$P(\nu) = \sqrt{3}\frac{e^3 B_\perp}{mc^2} F\left(\frac{\nu}{\nu_c}\right)$$

$$\nu_c = \frac{3}{4\pi}\gamma^2 \frac{eB_\perp}{mc}$$



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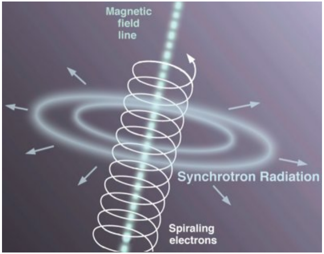

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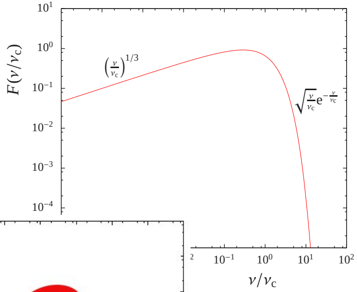
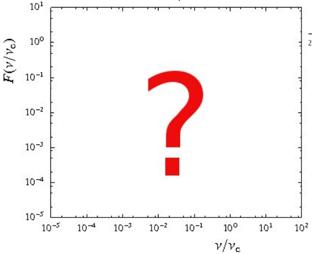
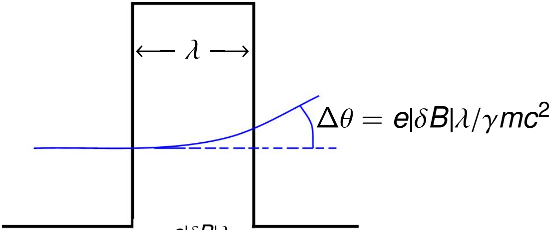




Radiation produced in scattering — simple picture

Consider a structure of width λ and field strength δB



Define strength parameter $a \equiv \frac{e|\delta B|\lambda}{mc^2}$: ratio of deflection angle to beaming angle

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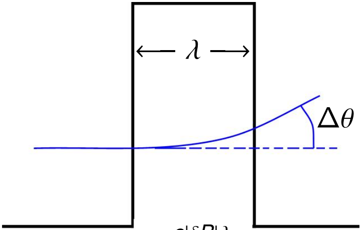

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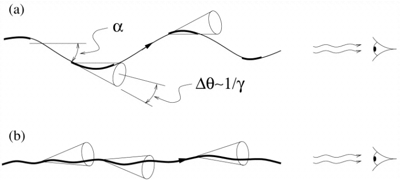
Maximum photon energy differs $a \gtrless 1$

$$\omega_{\max} = \begin{cases} 0.5\gamma^2 \frac{e\delta B}{mc} & \text{for } a > 1 \\ 0.5\gamma^2 c/\lambda & \text{for } a < 1 \end{cases}$$

(Landau & Lifshitz, C.T.O.F.)



Medvedev '00

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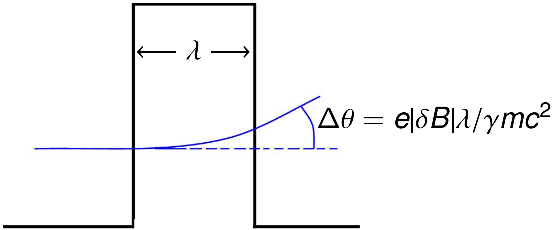

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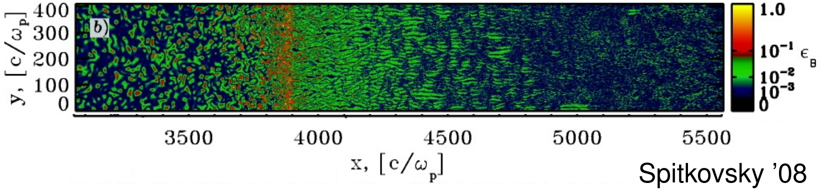
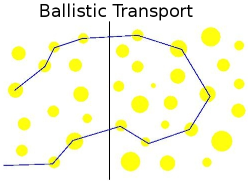


Spitkovsky '08

Maximum energy — 2 transport regimes

Ballistic: $a < \gamma$

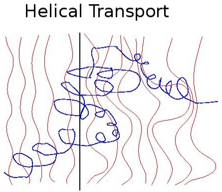
- particle deflections small $\Delta\theta = a/\gamma < 1$
- $N_{\rm scatt} \sim (\gamma/a)^2$ scatterings to return from downstream to upstream with $\frac{\Delta\gamma}{\gamma} \approx -\frac{2a^2e^2\gamma}{3mc^2\lambda}$ at each scattering



$$\gamma_{\rm B} < \left(\frac{3mc^2\lambda}{2e^2}\right)^{1/3}$$

Helical: $a > \gamma$

- cross field diffusion essential $\delta B/B_0 > 1$
- energy radiated constantly along trajectory



Bohm limit: $\gamma_{\rm H} < \left(\frac{3m^2c^4}{2e^3B}\right)^{1/2}$ (Achterberg et al. '01)

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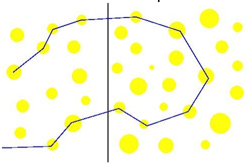

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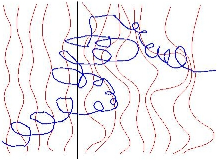
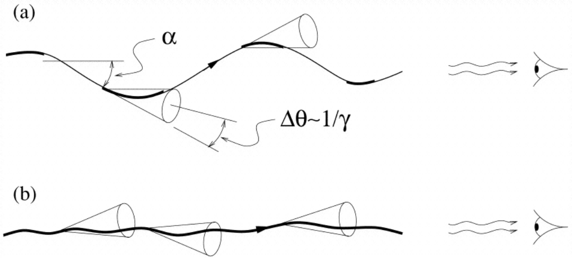


Bohm limit: $\gamma_{\rm H} < \left(\frac{3m^2c^4}{2e^3B}\right)^{1/2}$ (Achterberg et al. '01)

define critical strength parameter $a_{\rm crit} = \gamma_{\rm B}$

$$\gamma_{\max} = \begin{cases} a_{\rm crit} & \text{for } a < a_{\rm crit} \\ a_{\rm crit}\sqrt{a_{\rm crit}/a} & \text{for } a > a_{\rm crit} \end{cases}$$

Maximum photon energy — 2 regimes



synchrotron: $a > 1$

- transport can be ballistic or helical
- $\omega_{\rm max} \approx 0.5 a\gamma_{\rm max}^2 c/\lambda$

'jitter': $a < 1$

- particle motion is ballistic
- $\omega_{\rm max} \approx 0.5 \gamma_{\rm max}^2 c/\lambda$

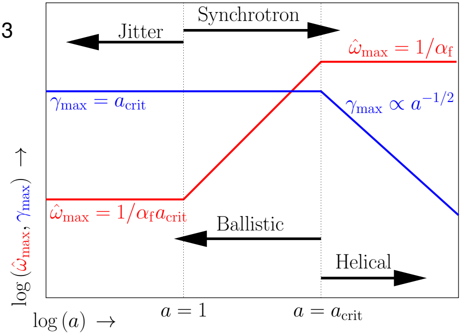
$$\hat{\omega}_{\rm max} \equiv \frac{\hbar\omega_{\rm max}}{mc^2} = \begin{cases} (\alpha_{\rm f} a_{\rm crit})^{-1} & a < 1 \\ a\,(\alpha_{\rm f} a_{\rm crit})^{-1} & 1 < a < a_{\rm crit} \\ \alpha_{\rm f}^{-1} & a > a_{\rm crit} \end{cases}$$

$\alpha_{\rm f}$ = fine structure constant ($\approx 1/137$)

Putting it all together

For fixed $a_{\rm crit}$

$$a_{\rm crit} = \left(\frac{3mc^2\lambda}{2e^2}\right)^{1/3}$$

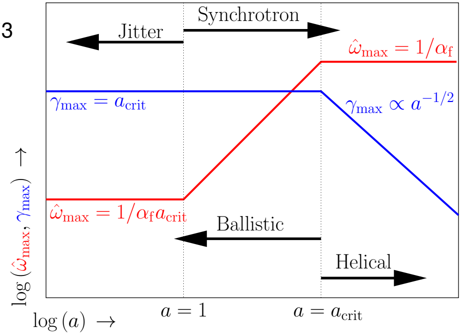


Kirk & BR '10

Putting it all together

For fixed a_{crit}

$$a_{\text{crit}} = \left(\frac{3mc^2\lambda}{2e^2}\right)^{1/3}$$



Kirk & BR '10

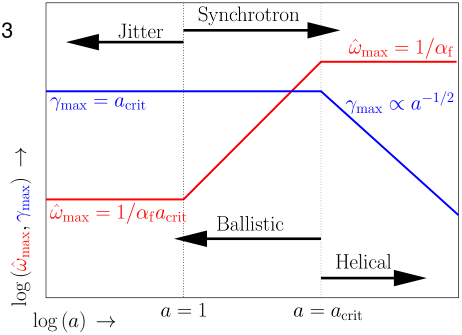
For Weibel mediated shocks $a_{\text{crit}} \approx 10^6 \Gamma^{1/6} \left(n/1\,\text{cm}^3\right)^{-1/6}$

For synch. maser inst. shocks $a_{\text{crit}} \approx 10^5 \Gamma^{1/3} \left(B/\text{mG}\right)^{-1/3}$

Putting it all together

For fixed a_{crit}

$$a_{crit} = \left(\frac{3mc^2\lambda}{2e^2}\right)^{1/3}$$



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Current PIC simulations suggest $a \sim \Gamma$ (Sironi & Spitkovsky '09)

$\hat{\omega}_{max} \sim \Gamma/\alpha_f a_{crit} \ll 1$

$\Rightarrow$ Difficult to produce γ-rays via synchrotron/jitter in GRBs

Is there a way around this limit?

1. Energy losses $\propto B^2\lambda$ (Larmor's formula)
2. scattering angles $\propto B\lambda$

Two populations of scatterers

1. radiation produced by short wavelength, strong B field structures
2. isotropisation from longer wavelength, weaker field structures

Maximum energy increases to

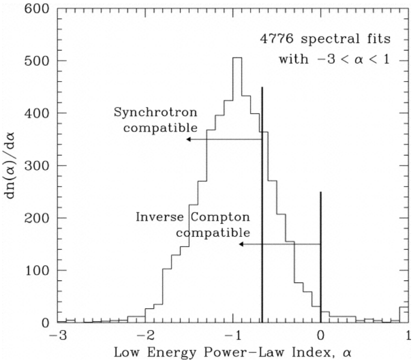
$$\hbar\omega_{\mathrm{max}} \sim \frac{\Gamma}{\alpha_{\mathrm{f}} a_{\mathrm{crit}}} \left(\frac{\langle \lambda_{\mathrm{iso}} \rangle}{\langle \lambda_{\mathrm{rad}} \rangle} \right)^{4/3} \ \mathrm{MeV}$$

Summary

- Fermi acceleration theory well established, but not complete
- PIC simulations show evidence of acceleration from scratch
  - In very weakly magnetised shocks
  - In subluminal magnetised shocks (rare in nature)
- theory is consistent with observations (spectral shape) in general but far from complete (maximum energy, turbulence, magnetised shocks)
- combining theory with observations will provide possibility of probing microphysics in these environments

Radiative signatures of Fermi acceleration

- Histogram of low energy photon index for BATSE detected GRBs ($\epsilon < h\nu_b$, ν_b=peak in νF_ν)
- $n(\epsilon) \propto \epsilon^\alpha$ ($F_\nu \propto \nu^{\alpha+1}$) Preece et al '00



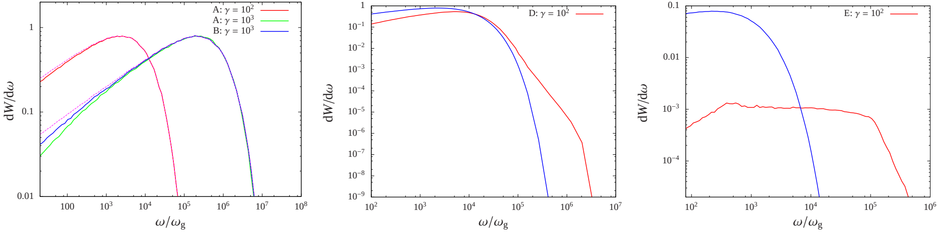
  - histogram peaks at $\alpha \approx -1$.
  - roughly 25% inconsistent with synchrotron emission

Emission spectra in turbulent fields

Numerically integrate particle trajectories in prescribed turbulent fields $W(\lambda) \propto \lambda^{\alpha}$, $\lambda_{\min} < \lambda < \lambda_{\max}$

Determine emission spectra from Liénard-Wiechert potentials

$a_{\max} \gg 1$ $\qquad$ $a_{\max} \sim 1$ $\qquad$ $a_{\max} \ll 1$



BR & Kirk '10

- low frequency spectral index can be harder than 1/3 in turbulent fields
- high frequency tails can be produced if sufficient energy in small *a* modes
- if distinguishable from source spectrum, observations can probe turbulence