7. The Crab Nebula Spectrum and its Physical Interpretation

Astrolab Landessternwarte Königstuhl

Max Camenzind & Jochen Heidt

March 4, 2024

Abstract

A new set of observations of the remnant of a tremendous stellar explosion gives astronomers a remarkable look at the dynamic relationship between the tiny Crab Pulsar and the vast nebula that it powers. Both, the nebula and the pulsar are bright sources of radiation at all wavelengths from radio to highest energies in the TeV range. The radiation is produced mainly by highly energetic particles accelerated by the rapidly rotating neutron star. These energetic particles spiral around magnetic field lines in the nebula and emit radiation via synchrotron and inverse Compton processes. By analyzing the global spectrum of the Crab nebula, we can learn a lot about the processes responsible for the radiation in quasars, for example.

1 Introduction

The Crab Nebula is the most famous and conspicuous known supernova remnant, an expanding cloud of gas created bye the explosion of a star as supernova which was observed in the year 1054 AD. The supernova was noted on July 4, 1054 A.D. by Chinese astronomers as a new or guest star, and was about four times brighter than Venus, or about mag -6. According to the records, it was visible in daylight for 23 days, and 653 days to the naked eye in the night sky. It was probably also recorded by Anasazi Indian artists (in present-day Arizona and New Mexico), as findings in Navaho Canyon and White Mesa (both Arizona) as well as in the Chaco Canyon National Park (New Mexico) indicate. In addition, Ralph R. Robbins of the University of Texas has found Mimbres Indian art from New Mexico, possibly depicting the supernova.

The nebula consists of the material ejected in the supernova explosion, which has been spread over a volume approximately 10 light years in diameter, and is still expanding at a very high velocity of about 1800 km/sec. The notion of gaseous filaments and a continuum background was photographically confirmed by Walter Baade and Rudolph Minkowski in 1930. The filaments are apparently the remnants from the former outer layers of the former star (the pre-supernova or supernova progenitor), while the inner, blueish nebula emits continuous light consisting of highly polarized synchrotron radiation, which is emitted by high-energy (fast moving) electrons in a strong magnetic field (check e.g. the VLT image of the Crab Nebula on the title page). This explanation was first proposed by the Soviet astronomer J. Shklovsky (1953) and supported by observations of Jan H. Oort and T. Walraven (1956).

At the center of the nebula is the Crab Pulsar, a neutron star remnant of the supernova which is roughly 22 km in diameter. It was discovered in 1968. The Crab Pulsar rotates once every 33 milliseconds, or 30 times each second. The most dynamic feature in the inner part of the nebula is the point where the relativistic pulsar wind slams into the surrounding material forming a shock front (Fig. 1). The shape and position of this feature shifts rapidly, with the equatorial wind appearing as a series of wisp-like features that steepen, brighten, then fade as they move away from the pulsar to well out into the main body of the nebula.

The Crab Nebula is often used as a calibration source in X-ray and γ-ray astronomy as it is very bright at these wavelengths. In addition, its flux density and spectrum are known to be constant, with the exception of the pulsar. The pulsar delivers a strong periodic signal that is used to check the timing of the X-ray detectors. In X-ray astronomy, 'Crab' and 'milliCrab' are sometimes used as units of flux density. Very few X-ray sources ever exceed one Crab in brightness.

2 Expansion and Distance of the Crab Nebula

We will now determine some properties of the Crab Nebula as well as the periods, dispersion and distance to some pulsars by solving the tasks listed below. The images/diagrams required for those tasks will be handled to you in printed form by the tutors. These are the *finder chart* for the Crab pulsar, the images of the Crab nebula from *January 19 1942* and from *February 27 1976*, the *Crab nebula spectrum* and the *Pulsar time series*. Alternatively, can can download them from the astrolab-web here.

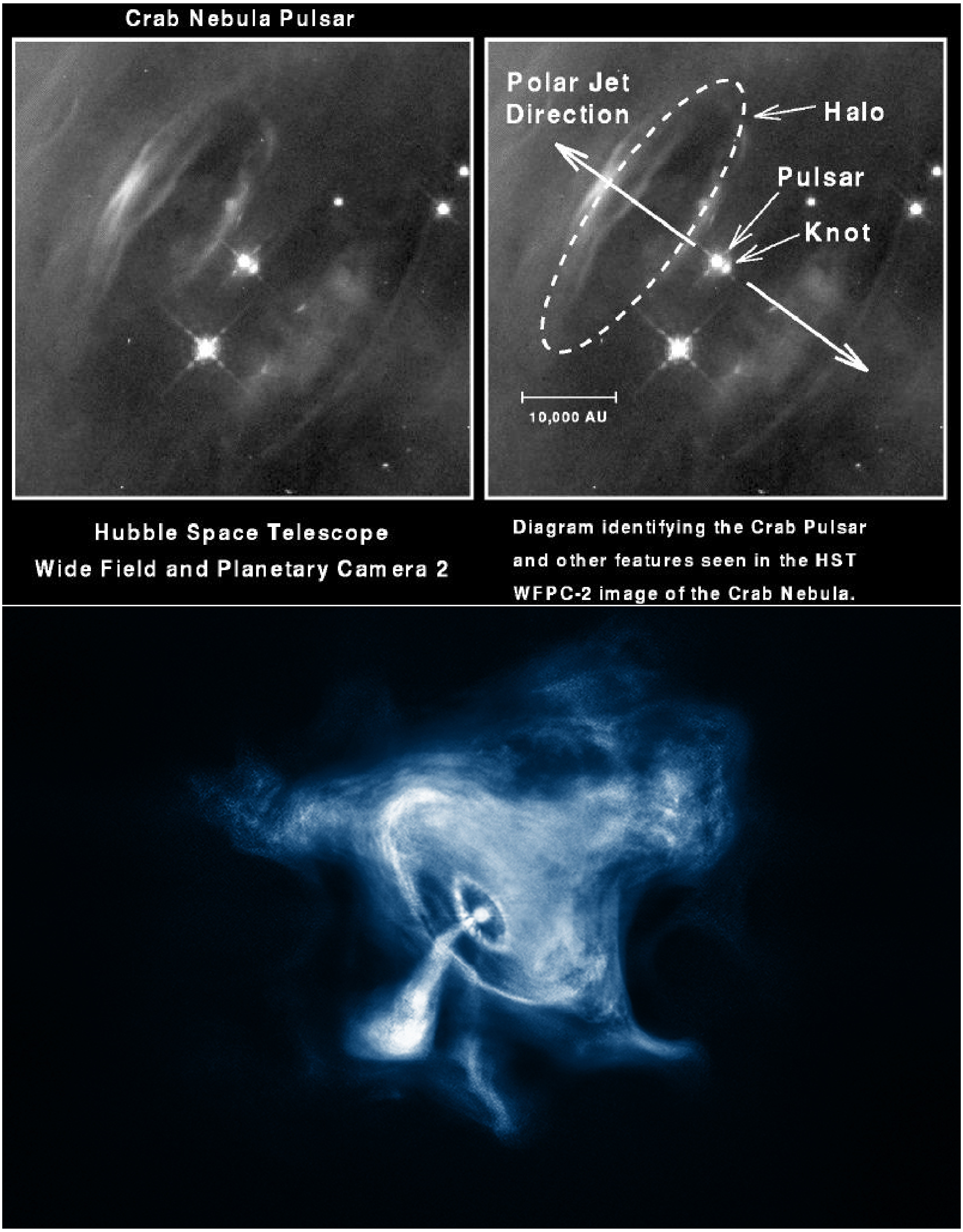

Figure 1: *Top) HST-image of the Crab Nebula. Bottom) Chandra-image of the Crab Nebula.*

Tasks and questions:

1. Determine the "kinematic" age of the Crab Nebula.
   The Crab Nebula is the expanding supernova remnant (SNR) of the "Chinese supernova" of the year 1054. You shall determine the proper motion $\mu = \frac{\Delta x}{\Delta t}$ for a number of knots of emission from images taken in 1942 and 1976 (time interval $\Delta t = 34$years). The linear shift will be measured with respect to the pulsar in the Crab Nebula. The scale on both images can be determined from the two marked stars, which are separated by $576''$. The pulsar in the Crab Nebula is the southern of the two stars in the center of the nebula. We assume that matter ejected by the supernova moves with a constant velocity; the gas ejected with the highest velocity has the largest distance to the central star. Using the measured angular velocity of each knot and the distance to the pulsar the "kinematic" age of the Crab Nebula and hence the year of the supernova explosion can be estimated. Check if your estimate corresponds (within the errors) to the dates found in Chinese sources.
   Material required: Images the Crab Nebula from 1942 and 1976 as well as an image showing the location of the pulsar.

2. Determine the distance of the Crab Nebula from the proper motion of knots of emission and the expansion velocity of the nebula along the line of sight.
   If one measures the expansion velocity of the nebula along the line of sight (radial velocity in km s^{-1}), the distance of the pulsar in the Crab Nebula can be determined from the angular velocity and the linear velocity. The expansion velocity can be determined from the splitting of the [O II] $\lambda\,3727$Å line in the enlarged spectrum (slit along the semi-major axis) of the Crab Nebula).
   Material required: Spectrum of the Crab Nebula taken along its semi-major axis.

3. Determine the periods of PSR 0809+74, PSR 0950+08 and PSR 0329+54
   From the recordings you can determine the pulsar periods of PSR 0809+74, PSR 0950+08 and PSR 0329+54. The highest accuracy can be achieved by using the widest separated pulses divided by the number of periods. Since the period is a characteristic quantity of a pulsar, it can be measured separately at three different frequencies.
   Material: Diagram of the radio signals of three pulsars at three different frequencies.

4. Determine the dispersion of PSR 0809+74, PSR 0950+08 and PSR 0329+54
   Radio pulses emitted at different frequencies by the pulsar move with different velocities through the interstellar medium due to interaction with free electrons. Pulses with a lower frequencies are more delayed by the free electrons and hence later detected (see also frequency dependence in the recordings). This effect is called "pulse dispersion". The delay is proportional to the number of electrons n [cm^{-3}] and the distance d [pc] of the pulsar. The quantity nd is called dispersion. The equation to calculate differences of travel time Δt for two frequencies f_1 and f_2 is

$$\Delta t = 4150 \cdot nd \cdot \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \tag{1}$$

   with Δt in s, n in cm^{-3}, d in pc and f_1,f_2 in MHz.
   Material required: Diagram of the radio signals of three pulsars at three different frequencies.

5. Determine the interstellar electron density from distance and dispersion of the pulsar in the Crab Nebula.
   The dispersion of the pulsar in the Crab Nebula is $nd = 56 cm^{-3}$ pc. Equation (1) can be used to

determine the electron density in the interstellar medium with known distance of the pulsar. The dispersion of the pulsar in the Crab Nebula is $nd = 56cm^{-3}$ pc. Determine the mean electron density using the distance derived in task 2 along the line of sight to the Crab Nebula.

6. Estimate the distance to PSR 0809+74, PSR 0950+08 and PSR 0329+54 with the dispersion. Equation (1) is often used for distance determination of pulsars. Assuming a mean electron density $n \approx 0.03\text{cm}^{-3}$ the distance of PSR 0809+74, PSR 0950+08 and PSR 0329+54 can be estimated.

3 Spectrum and Energy-Loss of the Crab Nebula

The emission of the Crab Nebula is of non-thermal origin, and the spectrum covers 20 orders of magnitude in energy space. The overall spectrum is typical for synchrotron emitters (supernovae remnants, radio galaxies, quasars and BL Lac objects). For the last 50 years, every new high-energy instrument in Astronomy has been calibrated with this object. In the last decades, new instruments have been built to measure the high-energy gamma-ray flux from the Crab Nebula (Compton Observatory, HEGRA, HESS, and many others).

3.1 Data sets

A selection of spectral data, which we will use in the following can by found in the Appendix and can be downloaded from the astrolab-WEB here. These are the tables 1-4. We divide these four data sets into low energy regime data from radio frequencies to X-ray energies, and in high energy data covering the range from MeV to TeV regime. *Note that you have to convert the high-energy photon fluxes to Janskys first before compiling. This is a somewhat tricky exercise. Bachelor and Master-students should give it a try, but if the do not succeed, they should talk to the tutor for help. PhD-students are expected to find that out by their own. On the astrolab-WEB you will also find a table (all 4 tables converted into one).*

Tasks and questions:

1. What is the meaning of the flux unit Jansky ?
2. Generate 5 different spectra in double-logarithmic form: a low-frequency flux spectrum S_ν and energy distribution νS_ν, a high-energy photon spectrum and high-energy energy distribution, and the global energy distribution νS_ν for the entire data set. Comment on these different spectra.
3. Some portions of the spectrum can be represented in terms of power-laws of the following form

$$S_\nu = S_0 \cdot (\nu/\nu_0)^{-\alpha} \ , \ \nu_0 \leq \nu \leq \nu_1 \tag{2}$$

where α is the spectral index, and S_0 the corresponding flux normalization. How many spectral power-laws are necessary to cover the entire spectrum ? Calculate the corresponding spectral indices by using a plot of the flux distribution.

4. Consider in particular the γ-spectrum and try to find an interpretation for the rather complicated behavior of the spectrum.
5. Find out the meaning of COMPTEL, EGRET, WHIPPLE, HEGRA and HESS.

3.2 The Energy-Loss of the Crab Nebula

You can use the plot for the energy distribution to estimate the total energy lost by the Crab Nebula. Remember that νS_ν is in units of Watt/m^{-2} . We can therefore integrate the spectrum

$$S = \int_0^\infty S_\nu d\nu = \int_0^\infty (\nu S_\nu)\ d\ \log\nu \simeq \sum_0^\infty (\nu S_\nu)_n\ (\Delta\log\nu)_n. \tag{3}$$

Tasks and questions:

1. Prepare a diagram of the energy distribution νS_ν.
2. Interpret the power spectrum. In which spectral ranges does the Crab Nebula emit most of its energy?
3. Calculate the total power S emitted by the Crab Nebula using equation 3. Compare this value with the solar constant $S_\odot$.
4. Use the distance d of the Crab Nebula to determine its total luminosity L. Give this value in solar units $L_\odot$.
5. Interpret of the infrared bump in the spectrum. Where does it come from?

3.3 Properties of the Synchrotron Emission

Monoenergetic relativistic electrons (and positrons) spiraling in a magnetic field cool by synchrotron emission with a characteristic frequency ν_c, which is related to the **cyclotron frequency** ν_B via

$$\nu_c \ \simeq\ 0.29\ \nu_0 \ \ \text{with}\ \ \nu_0 \ = \ \gamma_e^2\ \nu_B\ sin\chi. \tag{4}$$

γ_e is the Lorentz factor of the electron with energy $E_e = \gamma_e m_e c^2$, and χ is the pitch angle of the electron's helical motion in the magnetic field with field strength B. One finds then the relation

$$\nu_0 = 42\ \text{GHz}\ B_\perp \gamma_e^2 \tag{5}$$

where $B_\perp$ is the magnetic field perpendicular to the line of sight ($B_\perp$ is given in units of Tesla). The magnetic field in the inner part of the Crab nebula has been found to be of the order 30 to 50 nano-Tesla. Due to this synchrotron cooling the electrons loose energy with the rate

$$\dot{E}_e = \frac{4}{3}\ \sigma_T\ c\ \frac{B^2}{2\mu_0}\ \gamma_e^2. \tag{6}$$

The energy loss is proportional to the energy density in the magnetic field, and it scales with the square of the Lorentz factor γ_e. σ_T is the Thomson scattering cross-section for electrons. From this formula we derive the characteristic **cooling time** for electrons to $t_S = \gamma_e/\dot{\gamma}_e$. It is often assumed in astronomy that the pitch angles of synchrotron electrons are isotropically distributed. Under this assumption, the electron distribution in the Crab nebula depends on the location $\vec{x}$ in the nebula and on the Lorentz factor

$$n_e(\vec{x}, \gamma_e)\ d\gamma_e = n_0(\vec{x})\ N_e(\gamma_e) d\gamma_e. \tag{7}$$

The energy distribution of the electrons $N_e(\gamma_e)$ is typically a power-law, i.e. $N_e(\gamma_e) \propto \gamma_e^{-p}$ with the energy index p. The radiation flux observed in a telescope is therefore given by the line of sight integration of the local spectral emissivity $j_\nu(B_\perp, \gamma_e)$ over the entire volume of the nebula

$$S_\nu = \frac{1}{4\pi d^2} \int_{nebula} n_0(\vec{x}) \left[\int_0^\infty N_e(\gamma_e) j_\nu(B_\perp, \gamma_e) d\gamma_e \right] dV. \tag{8}$$

The integration of the synchrotron emissivity j_ν over the energy distribution N_e leads to a power-law in the observed spectrum, $S_\nu \propto \nu^{-\alpha}$, where the spectral index is related to the energy spectral index p via the relation

$$\alpha = \frac{p-1}{2} \tag{9}$$

This explains, why the observed spectrum is a sequence of power-laws.

Tasks and questions:

1. Why is the emission from the Crab nebula synchrotron emission (give two arguments).
2. Determine the typical Lorentz factors γ_e for the electrons which emit radio, optical, X- and γ-rays, respectively.
3. Determine the typical cooling times t_S for these electrons (radio, optical, X- and γ-rays). What do we learn from these numbers ?
4. Give arguments for the formula 8 by considering the radiation transport equation.
5. Find an interpretation for the TeV-spectrum of the Crab nebula.
6. What is inverse Compton emission? Which parameters determine the typical energy E_c emitted by inverse Compton processes (search in the literature)? Look for other sources, where inverse Compton emission is important.

3.4 The Energy Source of the Crab Nebula

So far we have found that the continuum emission of the Crab nebula is due to synchrotron losses of relativistic electrons (and positrons), which have to be refueled by the central Pulsar. The only energy reservoir of a rotating neutron star is its rotational energy, which can be derived from the observed rotational period $P = 33.3$ ms and the moment of inertia of the neutron star, $I_* \simeq 0.4 M_* R_*^2$. In addition, we know the braking of the rotation from observations to be $\dot{P} = 4.22 \times 10^{-13}$ s/s from pulsar timing.

Tasks and questions:

1. Calculate the rotational energy E_{rot} of the Crab Pulsar and from this its energy-loss $\dot{E}_{rot}$. Compare with your number for the total luminosity L of the Crab nebula.
2. The observed total luminosity L must be smaller than the rotational energy-loss $\dot{E}_{rot}$. Where is the rest of the energy?

3.5 The Inner Structure of the Crab Nebula

The Crab Nebula is the archetypal filled-center supernova remnant, or **plerion**. A central pulsar powers each filled-center supernova remnant. Thus, the inner nebula of a plerion is particularly interesting, since it is the site of conversion of pulsar-supplied energy into synchrotron-emitting electrons. The Crab Pulsar generates a relativistic wind which creates a cavity of 10 arcsec around the pulsar (see HST and Chandra images, Scargle's optical hole). At this distance, the pulsar wind is shocked and particles get accelerated to even higher energies. This structure is seen as a torus-like ring around the Pulsar, since the wind is not spherically symmetric, but more concentrated towards the equatorial plane. The inner Crab ring is one light year in diameter; in Vela it is 0.1 light year. Chandra images revealed, for the first time, an X-ray inner ring within the X-ray torus, the suggestion of a hollow-tube structure for the torus, and X-ray knots along the inner ring and perhaps along the inward extension of the X-ray jet.

References

[1] F. Aharonian et al. (HEGRA Coll.) 2004: The Crab Nebula and Pulsar between 500 GeV and 80 TeV: Observations with the HEGRA stereoscopic air Cherenkov telescopes, ApJ 614, 897
[2] J. Kuiper et al. 2001: The Crab pulsar in the 0.75-30 MeV range as seen by CGRO COMPTEL, A&A 378, 918; astro-ph/0109200
[3] http://cossc.gsfc.nasa.gov/docs/cgro/index.html
[4] http://www.mpi-hd.mpg.de/hfm/HESS/HESS.html
[5] http://heasarc.gsfc.nasa.gov/docs/xte/XTE.html

4 Appendix: Data Sets

Table 1: *Spectral fluxes in the low-frequency regime. The X-ray data are from RXTE* [5].

Frequency/Wavelength	**Spectral Flux**
100 MHz	2080 Jy
1 GHz	1040 Jy
22 GHz	411 Jy
250 GHz	204 Jy
0.3 mm	120 Jy
100 μ m	184 Jy
60 μ m	210 Jy
25 μ m	67 Jy
12 μ m	37 Jy
2.2 μ m	9.12 Jy
1.6 μ m	9.33 Jy
740 nm	5.25 Jy
470 nm	4.26 Jy
330 nm	1.91 Jy
155 nm	1.37 Jy
2.463 keV	1.351 ph/cm^2 s keV
2.866 keV	0.9778 "
4.210 keV	0.4412 "
6.069 keV	0.2052 "
8.124 keV	0.1135 "
10.43 keV	0.0677 "
15.07 keV	0.0317 "
22.37 keV	0.0143 "
44.62 keV	0.0041 "
67.75 keV	0.0011 "

Table 2: *COMPTEL spectra of the Crab nebula [2].*

Energy window [MeV]	Nebula Photon Flux [ph/cm^2 s MeV]
0.75 - 1.00	2.585 $\pm$0.089 $\cdot 10^{-03}$
1.00 - 1.25	1.563 $\pm$0.054 $\cdot 10^{-03}$
1.25 - 1.50	1.127 $\pm$0.043 $\cdot 10^{-03}$
1.50 - 2.00	0.617 $\pm$0.020 $\cdot 10^{-03}$
2.00 - 2.50	0.306 $\pm$0.014 $\cdot 10^{-03}$
2.50 - 3.00	0.217 $\pm$0.010 $\cdot 10^{-03}$
3.0 - 4.00	1.312 $\pm$0.055 $\cdot 10^{-03}$
4.00 - 6.00	0.613 $\pm$0.022 $\cdot 10^{-03}$
6.00 - 8.00	0.284 $\pm$0.014 $\cdot 10^{-03}$
8.00 - 10.0	1.637 $\pm$0.082 $\cdot 10^{-03}$
10.0 - 15.0	0.734 $\pm$0.033 $\cdot 10^{-03}$
15.0 - 30.0	0.201 $\pm$0.013 $\cdot 10^{-03}$

Table 3: *EGRET data [3]*

Energy	**Photon Flux [ph/cm^2 s MeV]**
50 MeV	1.5 $\cdot 10^{-07}$
120 MeV	9.0 $\cdot 10^{-09}$
200 MeV	1.5 $\cdot 10^{-09}$
400 MeV	2.0 $\cdot 10^{-10}$
800 MeV	3.0 $\cdot 10^{-11}$
2.0 GeV	1.2 $\cdot 10^{-11}$
8.0 GeV	3.0 $\cdot 10^{-12}$

Table 4: *HEGRA data for the Crab nebula [1]*

Energy [**TeV**]	**Photon Flux** [**ph/cm^2 s MeV**]
0.365	$1.97 \pm 1.17 \cdot 10^{-10}$
0.487	$1.76 \pm 0.24 \cdot 10^{-10}$
0.649	$8.78 \pm 0.53 \cdot 10^{-11}$
0.866	$4.02 \pm 0.13 \cdot 10^{-11}$
1.155	$1.87 \pm 0.09 \cdot 10^{-11}$
1.540	$9.05 \pm 0.26 \cdot 10^{-12}$
2.054	$4.51 \pm 0.12 \cdot 10^{-12}$
2.738	$2.16 \pm 0.07 \cdot 10^{-12}$
3.652	$9.33 \pm 0.36 \cdot 10^{-13}$
4.870	$4.18 \pm 0.20 \cdot 10^{-13}$
6.494	$1.93 \pm 0.12 \cdot 10^{-13}$
8.660	$1.02 \pm 0.07 \cdot 10^{-13}$
13.335	$3.28 \pm 0.31 \cdot 10^{-14}$
23.714	$5.28 \pm 0.70 \cdot 10^{-15}$
42.170	$1.10 \pm 0.25 \cdot 10^{-16}$
74.989	$2.05 \pm 1.01 \cdot 10^{-16}$