Determining the Rotation Curve of the Milky Way

8.1 Goal

Determine the rotation curve $v(r)$ of our galaxy within the solar orbit from radio observations of H-I regions.

8.2 Materials

- Task manual (this document).
- H-I map and profiles from Burton, Astron. & Astrophys. Suppl. 2, 261 (1970).

8.3 Literature

- H.-H. Voigt: Abriss der Astronomie, Kap. IX, 4.3 (brief overview of quantities and methods).
- A. Unsold, B. Baschek: Der Neue Kosmos, Kap. 5.2.
- Gunn et al.: Astron. J. 84, 1181 (1978) (calculation of the rotation curve using the same method).
- Knapp et al.: Astron. J. 83, 1585 (1978), (calculation of the rotation curve outside the solar orbit using a different method).
- P. Schneider: Extragalactic Astronomy and Cosmology (2015).

8.4 Experimental Preparation

The determination of the rotation curve $v(r)$ of our galaxy is of fundamental importance for the development of dynamic models of its structure and, therefore, for our understanding of its properties with respect to an external observer. Several methods are available for determining the rotation law, all of which are based on the principle of measuring relative velocities between the Sun and objects that closely follow the mean rotation of the galaxy. Differences exist in the reduction method of the measurements as well as the measurement procedures. Since we observe from a coordinate system that is moving with (in first approximation, uniformly), and whose proper motion is linked to the relationship to be determined, we must know the location (and therefore also the distance!) of the galactic center **and** of the observed object in order to calculate the rotational velocity with respect to an inertial system at the galactic center.

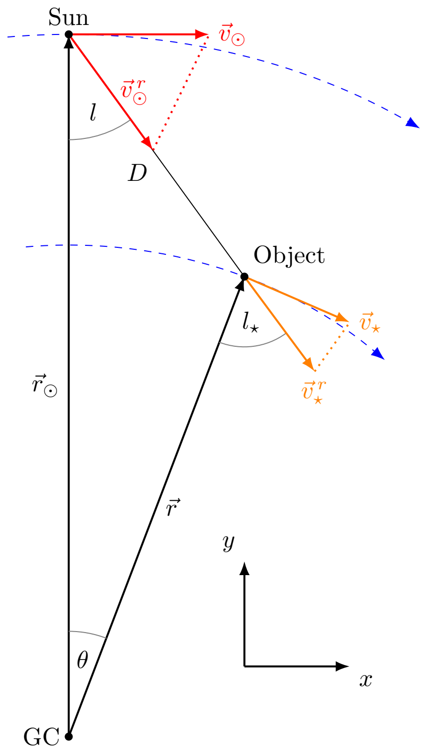

Figure 1: Geometry of the rotation of objects around the centre of the galactic centre (GC).

The geometric relationships are shown in Figure 1 for a point-like object. Assuming that the Sun and star describe strictly circular orbits, the rotation law can be calculated as follows: The measured radial velocity Δv^r follows from the difference between the radial components of the velocity vectors (the proper motion perpendicular to $\vec{v}^r$ is generally too imprecisely known):

$$\Delta v^r = |\vec{v}*^r| - |\vec{v}\odot^r| = v(r)\sin l_* - v_\odot \sin l, . \tag{1}$$

Application of the sine law eliminates the unknown l_*:

$$\sin l_* = \frac{r_\odot}{r} \sin l, . \tag{2}$$

This leads to the following by inserting into Equation (1):

$$\Delta v^r = r_\odot \left(\frac{v(r)}{r} - \frac{v_\odot}{r_\odot} \right) \sin l\,. \tag{3}$$

Unfortunately, the quantity r, i.e., the distance of an object to the galactic center, is generally not known. However, for stars or star clusters, a distance D can usually be given, so that, according to the cosine law for r, we have:

$$r^2 = D^2 + r_\odot^2 - 2Dr_\odot \cos l,. \tag{4}$$

The determination of the distance is in most cases of insufficient accuracy, or the motion of individual objects deviates too much from the average galactic rotation.

While equation (3) does not require any restrictions on the shape of $v(r)$, it is possible to represent r as a function of l by selecting a special class of functions $v(r)$ that correspond to a plausible model of the mass distribution of the galaxy. This is done as follows: In an observation direction l, we see different radial components of the respective circular orbit velocity along the line of sight. The radial component of the observed object has a value according to equation (3):

$$v_*^r = \frac{r_\odot}{r} v(r) \sin l,. \tag{5}$$

The measured velocity component Δv^r differs from this only by a constant summand for fixed l. So Δv^r is maximal when v_*^r is maximal:

$$v_*^r = \max\left(\frac{v(r)}{r}\right),. \tag{6}$$

If $v(r)$ does not increase too strongly outward, the position of the maximum will be the one with the smallest r. It is given by:

$$r_{\text{min}} = r_\odot \sin l \tag{7}$$

for $l_* = \pi/2$.

The set of all tangential points for l from $-90°$ to $90°$ is therefore on a circle through the locations of the Sun and the galactic center according to the theorem of Thales (see Figure 2). Substituting equation(7) into (3) for the maximum relative velocity yields an expression for $v(r)$ that does not depend explicitly on the distance of observed objects:

$$\begin{aligned} \Delta v^{\text{r, max}} &= v(r_{\text{min}}) - v_\odot \sin l \\ \Rightarrow v(r_{\text{min}}) &= v_\odot \sin l + \Delta v^{\text{r, max}}. \end{aligned} \tag{8}$$

The equations still contain the unknown quantities $r_\odot$ and $v_\odot$. The quantity $r_\odot$ only changes the scaling of the r-axis and therefore has no influence on the curve itself. $v_\odot$ fixes the outer end of the rotation curve (the inner end remains practically unaffected) and thus affects the curve's course. $r_\odot$ must be determined in any case independently of the $\Delta v^{\text{r, max}}$ measurement. $v_\odot$ is linked to the "differential rotation" determined from local velocity and distance measurements through:

$$\frac{v(r)}{r} - \frac{v_\odot}{r_\odot} = \omega(r) - \omega_\odot \approx \frac{\delta\omega}{\delta r} D \cos l \tag{9}$$

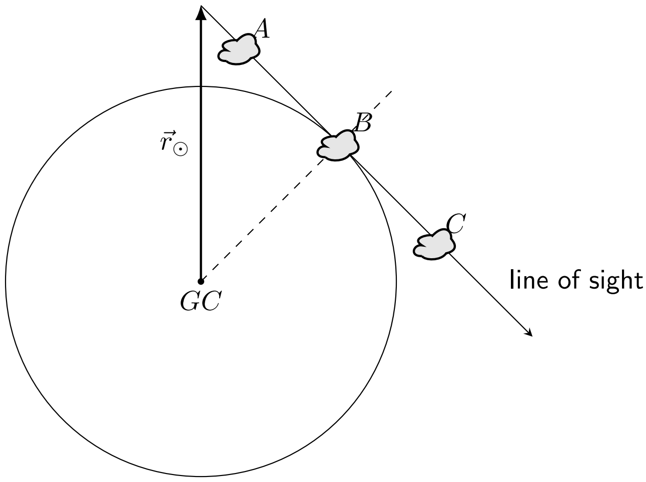

Figure 2: Objects in the line of sight.

and is determined independently from it. We use the standard values of the IAU from 1985 with $r_\odot = 8{,}5\,\mathrm{kpc}$ and $v_\odot = 220\,\mathrm{km\,s^{-1}}$.

Very suitable objects for a measurement using the described method are HI clouds. The 21 cm line emitted by them allows for high-precision velocity measurements, they are sufficiently evenly distributed over large areas of the galaxy, and follow the galactic rotation well. Since radiation in the radio range penetrates the interstellar medium almost absorption-free, we can also see and measure the tangential point for (almost) all l. Such measurements by Burton (1970) are available in the form of profiles and a map. For each galactic longitude l (in steps of 0,5°), the signal intensity of the 21 cm line is plotted against its Doppler shift and directly converted into intensities $I(\Delta v^r)$. The maximum velocity at the tangential point $\Delta v^{\mathrm{r,\,max}}$ corresponds to the right edge of the respective profile curve.

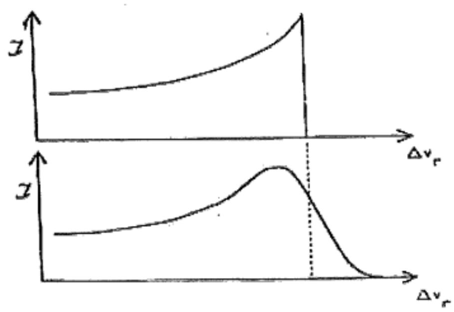

Figure 3: Tangential point as shown in case of Doppler broadening.

One difficulty in determining $\Delta v^{\mathrm{r,\,max}}$ is that the HI lines are broadened due to thermal motion, among other factors. For δ-shaped line profiles, a diagram in the form of Figure 3 above would be expected. The curve of the same emission with Gaussian broadening is shown below it. As can be seen, the point of theoretical maximum velocity is then somewhere at the right-side tail of the Gaussian and not at its maximum. The location of this point is somewhat arbitrary. The profile curves are accompanied by four examples of theoretically broadened profile curves, in which the correct value for $\Delta v^{\mathrm{r,\,max}}$ is marked by an arrow. This is intended to provide a basis for experience in estimating the correct maximum velocity.

8.5 Warm-up Questions

a) Under what conditions is Δv^r maximal when r is minimal? What does this mean for a spherically symmetric assumed mass distribution of the galaxy?

b) Besides the circular orbital velocity, what other physical quantitie determine the shape of the profile curves? What possible effects can be expected on the result?

c) How can $r_\odot$ and $v_\odot$ be determined?

d) What is meant by antenna temperature?

8.6 Experimental Procedure

a) Consider the profile curves of the intensities over Δv^r and the map of the intensities over $l \cdot \Delta v^r$. Compare the profiles and the map to determine which peaks at small galactic longitudes belong to the tangential point.

b) Read off the value of Δv^r for descending l at $l = 90°$. Pay attention to the position of the tangential point on the right-side Gaussian tail (see above). Proceed in steps of 2°. Up to what l can one trust that the measured velocities actually correspond to the velocity of HI clouds at the tangential point? Can you justify your assumption?

   Note: Use a suitable graphics program for reading off, or if you use a printout, use a ruler.

c) Convert the measured values to $v(r)$ using Equation (8) and graphically represent the function (e.g., in Python).

d) Discuss the course of the curve qualitatively with regard to the density distribution in the galaxy and with respect to perturbations.

e) At $r = 8{,}5\,\mathrm{kpc}$, your curve probably does not smoothly transition to the assumed value of $220\,\mathrm{km\,s^{-1}}$ for $v_\odot$. What do you attribute this inconsistency to?

f) Calculate $\omega(r_\odot)$ and thus the time for one orbit of the Sun around the galactic center. Estimate the total mass of the galaxy within the Sun's orbit assuming a spherically symmetric mass distribution. Compare your estimate with current reference values.